Deconvolution for Observational Asteroseismology

J. M. Joel Ong[1 2 *] Yaguang Li[1] Daniel Hey[1]

[1]Institute for Astronomy, University of Hawai'i; [2]NASA Hubble Fellow

[*]joelong@hawaii.edu

Modern asteroseismic data sets have **negligible timing errors**, as well as **well-characterised noise statistics**.

These allow us to apply **nonblind deconvolution** techniques from the image-processing literature.

Motivation: Window Functions in Gapped Time Series

Asteroseismic time series are not always continuous:

- Ground-based single-site observations suffer from **daily gaps**, even if data can be taken for consecutive nights.
- Repeat-visit missions like the *TESS* CVZs suffer from **annual gaps** between visits.

Gaps in the time domain yield **sidelobes in the frequency domain**, which interfere with mode identification and frequency fitting.

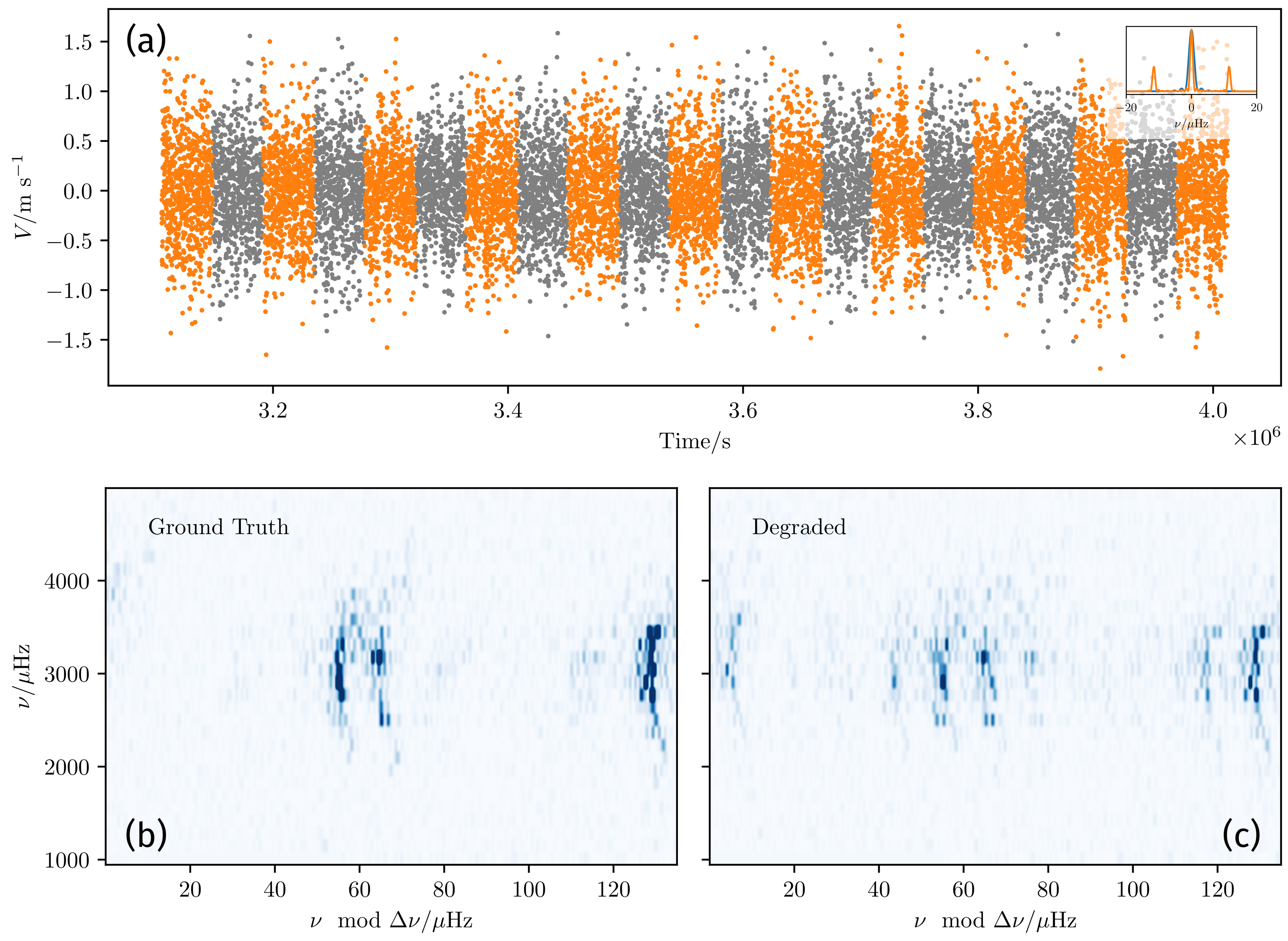

Figure 1: Numerical demonstration of window-function sidelobes. We artificially degrade MDI radial-velocity time series, shown in **(a)**; data points shown in gray are deliberately omitted when computing power spectra, resulting in a window function possessing sidelobes. This window function is illustrated in the inset panel (orange), compared with one of a continuous time series with equivalent duration (blue). Compared to the ground-truth power spectrum in **(b)**, the power spectrum of the gapped time series in **(c)** exhibits spurious peaks. In this case, the gaps are chosen to be separated such that the sidelobe spacing coincides with the small separation.

Positive-Definite Deconvolution

Deconvolution of Fourier amplitudes by division in the time domain is untenable by construction — the window function goes to zero and all information in gaps is lost. However,

- **Timing measurement errors are negligible**: the window function is essentially known perfectly. Given a series of exposures indexed by integers k, each with midpoint time t_k and of duration T_k, the transfer function of the unitary Fourier transform of that time series can be computed analytically by

$$W(\omega) = \frac{\left|\sum_k e^{i\omega t_k} T_k \mathrm{sinc}\left(\frac{\omega T_k}{2}\right)\right|^2}{\pi \sum_k T_k}. \qquad (1)$$

- Suppose the true underlying signal were to comprise of intensity fluctuations I_i from different modes, as well as some noise process N. If we suppose these $\{I_i\}$ and the noise are **independent stationary stochastic processes,** so that e.g. $\langle I_i I_j \rangle = 0$, then the expectation value of the power spectrum is such that

$$\langle P \rangle = \left\langle \left| \left(\sum_i \hat{I}_i + \hat{N} \right) * \hat{W} \right|^2 \right\rangle = \left\langle \left(\sum_i \left|\hat{I}_i\right|^2 + |\hat{N}|^2 \right) * \left|\hat{W}\right|^2 \right\rangle. \qquad (2)$$

  Thus, we may deconvolve the PSD against $|\hat{W}|^2$, rather than deconvolving the complex Fourier amplitudes against $\hat{W}$.

Notably, time-domain white noise is additive *before*, rather than *after*, convolution against frequency-domain window function $\implies$ **no noise amplification**!

Unlike deconvolution against the complex Fourier amplitudes, **the true deconvolved power spectrum must be positive semidefinite**. This allows us to use algorithms that suppress signed ringing artifacts.

Numerical Schemes

We implement two distinct iterative numerical schemes:

- **Richardson-Lucy Deconvolution**: Iterative relaxation to maximum-likelihood estimator for deconvolved power spectrum. Our implementation uses χ^2-2dof statistics, unlike the MSE metric ordinarily used for image denoising.
- **Gold Deconvolution**: Boosted solutions by Neumann series to the deconvolution problem expressed as a Volterra integral equation, with variable relaxation. This technique has historically been extensively used in γ-ray spectroscopy.

Our *jax*-accelerated implementation can be found on GitHub: *https://github.com/parallelpro/maemae*

Illustrative Results

It works!

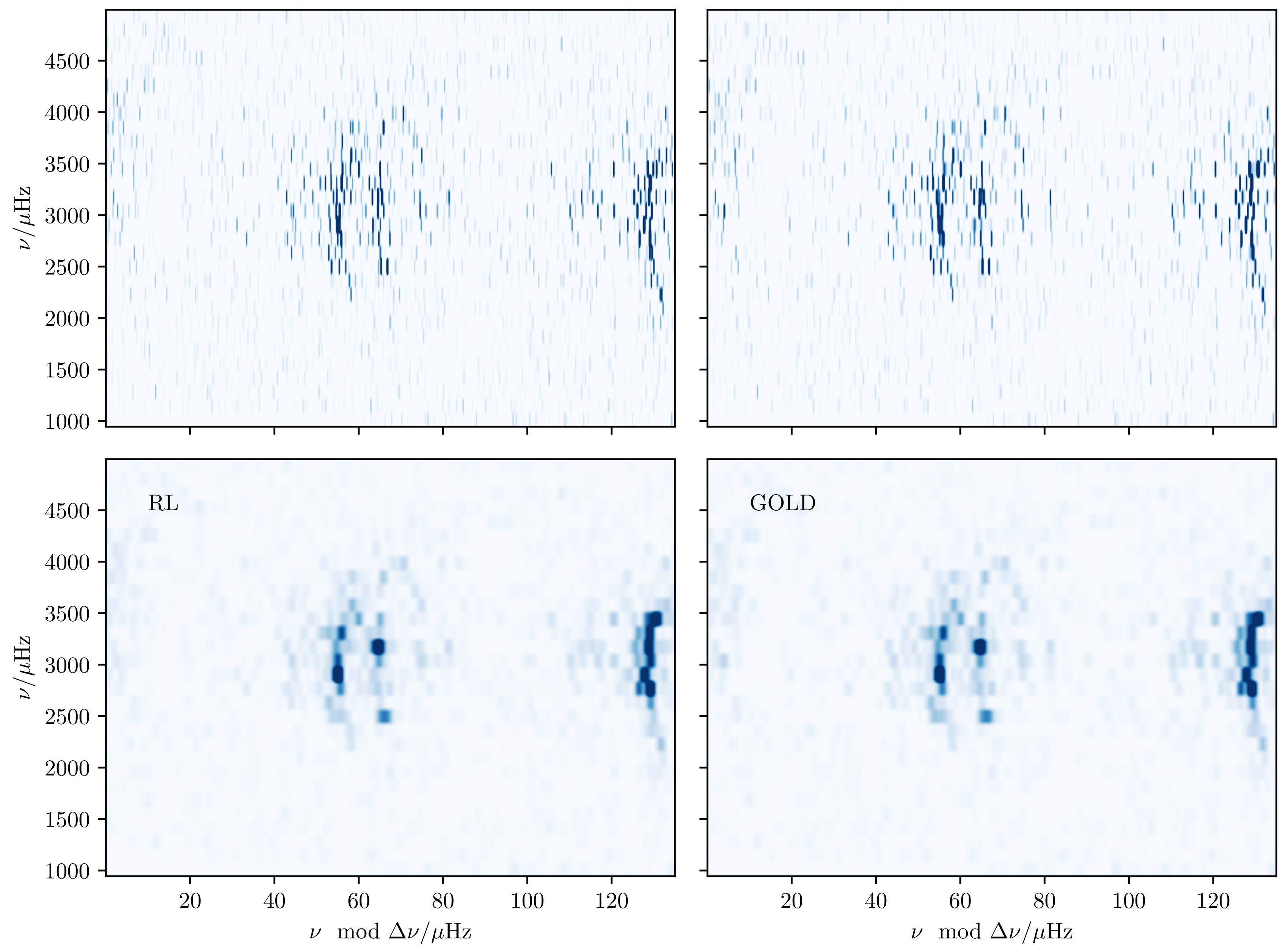

Figure 2: We successfully deconvolved the power spectrum shown in fig. 1(c). The left column of figures shows results produced by Richardson-Lucy deconvolution, and the right column shows those from Gold deconvolution. The upper row of figures shows the raw results of the deconvolution procedure, while the lower row shows results at the effective resolution of an equivalent continuous time series.

- Both of our deconvolution schemes yield results that (qualitatively) agree.
- Superresolution is spurious (exact deconvolution would have yielded δ-functions) but can be ameliorated with "reconvolution" step against effective window function, a la Bolton & Schlegel (2010)

Key Takeaways

- Sidelobes not *completely* suppressed, but good enough for mode identification!
- Already used this in production — e.g. Li et al. (2025); Kjeldsen et al. (in review)
- Forthcoming paper to examine systematics in other tasks (e.g. rotational splittings, classical pulsators) via hare-and-hounds exercise.

References

Bolton, A. S., & Schlegel, D. J. 2010, PASP, 122, 248, doi: *10.1086/651008*

Bradbury, J., Frostig, R., Hawkins, P., et al. 2018, JAX: composable transformations of Python+NumPy programs, 0.3.13. *http://github.com/google/jax*

Kjeldsen, H., Bedding, T. R., Li, Y., et al. in review, arXiv e-prints, arXiv:2506.00493, doi: *10.48550/arXiv.2506.00493*

Li, Y., Huber, D., Ong, J. M. J., et al. 2025, ApJ, 984, 125, doi: *10.3847/1538-4357/adc737*