Resolving an Asteroseismic Catastrophe: What do Red Giant Small Separations Signify?

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We derive expressions for small separations that **remain usable** on the red giant branch, where existing ones **fail catastrophically**.

These **allow us to interpret** newly discovered observational features of C-D diagrams.

Motivation: An Asteroseismic Catastrophe

The **small frequency separation** $\delta\nu_{02}(n) = \nu_{n,\ell=0} - \nu_{n-1,\ell=2}$ is used, in analysing main-sequence p-mode pulsators, as a diagnostic of interior structure. This is because the separation ratio

$$r_{02}(n) = \frac{\delta\nu_{02}(n)}{\Delta\nu_1(n)} = \frac{\nu_{n,\ell=0} - \nu_{n-1,\ell=2}}{\nu_{n,\ell=1} - \nu_{n-1,\ell=1}} \sim \frac{1}{\pi}\left(\delta_2(\omega) - \delta_0(\omega)\right), \tag{1}$$

depends only on the **inner phase functions** $\delta_\ell(\omega)$ — see e.g. Roxburgh (2005).

On the main sequence, this relates to **interior structure** as (e.g. Tassoul, 1990)

$$r_{\ell,\ell+2} \sim -\frac{2\ell+3}{\pi^2\nu}\left[\int_0^R \frac{1}{r}\frac{\mathrm{d}c_s}{\mathrm{d}r}\mathrm{d}r - \frac{c_s(R)}{R}\right] + \mathcal{O}\left(\frac{1}{\nu^2}\right). \tag{2}$$

However, this expression **does not work on the red giant branch!**

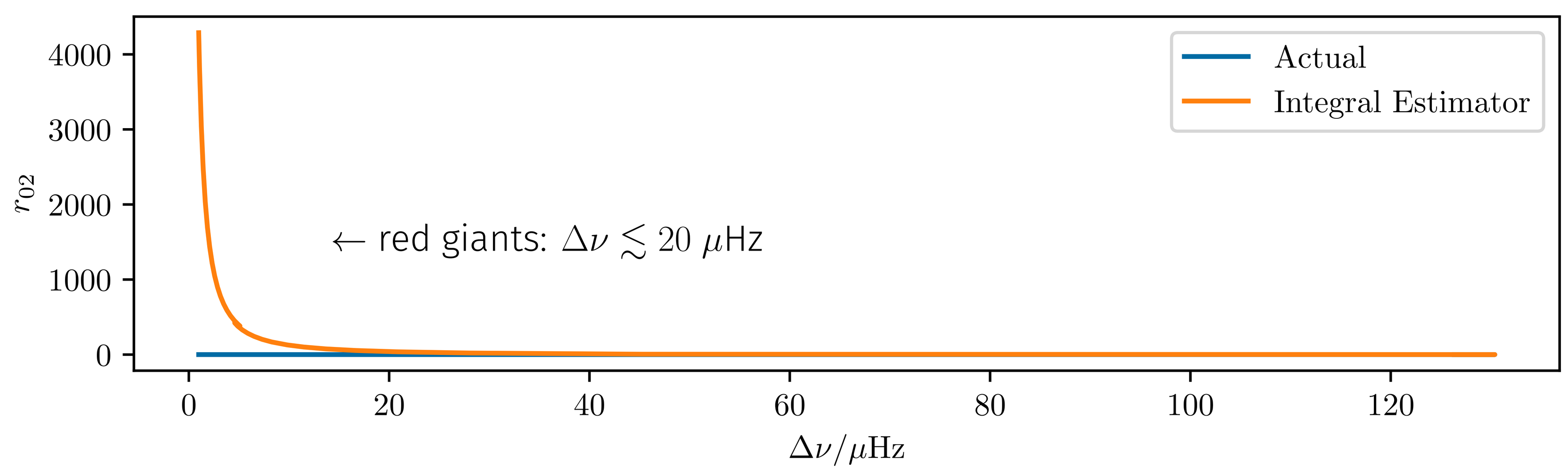

Figure 1: Small separation ratios r_{02} for an illustrative MESA evolutionary track. Values computed with mode frequencies (averaged over modes near $\nu_{\max}$) are shown in blue, while values computed using the integral expression of Tassoul (1990), eq. (2), are shown in orange. The actual small separation ratios are all much less than unity.

Clearly, eq. (2) fails when applied to RGB structure models.

- How do we resolve this apparent catastrophe?
- Conversely: what do the small separations in red giants actually tell us?

Revised Asymptotic Analysis

Adopting the notation of Roxburgh & Vorontsov (1994), the wavefunctions are parameterised as

$$\xi_r \sim a_\ell(\omega, t) j_\ell(\omega t - \delta_\ell(\omega, t)), \text{ with } \delta_\ell(\omega) \equiv \lim_{t\to T} \delta_\ell(\omega, t); \tag{3}$$

$t(r) = \int_0^r (1/c_s)\mathrm{d}r$ is the acoustic radius and j_ℓ is the first spherical Bessel function at degree ℓ.

Equation (2) comes from asymptotic approximation to these Bessel functions at large argument, s.t.

$$\delta_\ell(\omega) + \frac{\pi}{2} \sim \arctan\left[\frac{1}{\omega}\left\{A_0 + \ell(\ell+1)\left(A_\ell - \frac{1}{2T}\right)\right\}\right] + \arctan\left[\frac{\ell(\ell+1)}{2\omega T}\right] + \mathcal{O}\left(\frac{1}{\omega^2}\right); \tag{4}$$

here, A_ℓ is the integral of eq. (2), and

$$A_0 \sim \frac{1}{2}\int_0^T \left(4\pi G\rho - N^2\right)\,\mathrm{d}t. \tag{5}$$

- The catastrophe of fig. 1 results from the **failure of a small-angle approximation**.

In main-sequence stars, eq. (2) is obtained where $A_0 \ll 2\pi\nu_{\max}$. In red giants, where $A_0 \gg 2\pi\nu_{\max}$, we instead obtain

$$\boxed{r_{02,\mathrm{asy}} \sim \frac{1}{\omega T} \sim \frac{\Delta\nu}{\nu_{\max}}}. \tag{6}$$

(but the full expression describing transition between the two regimes is more complicated...)

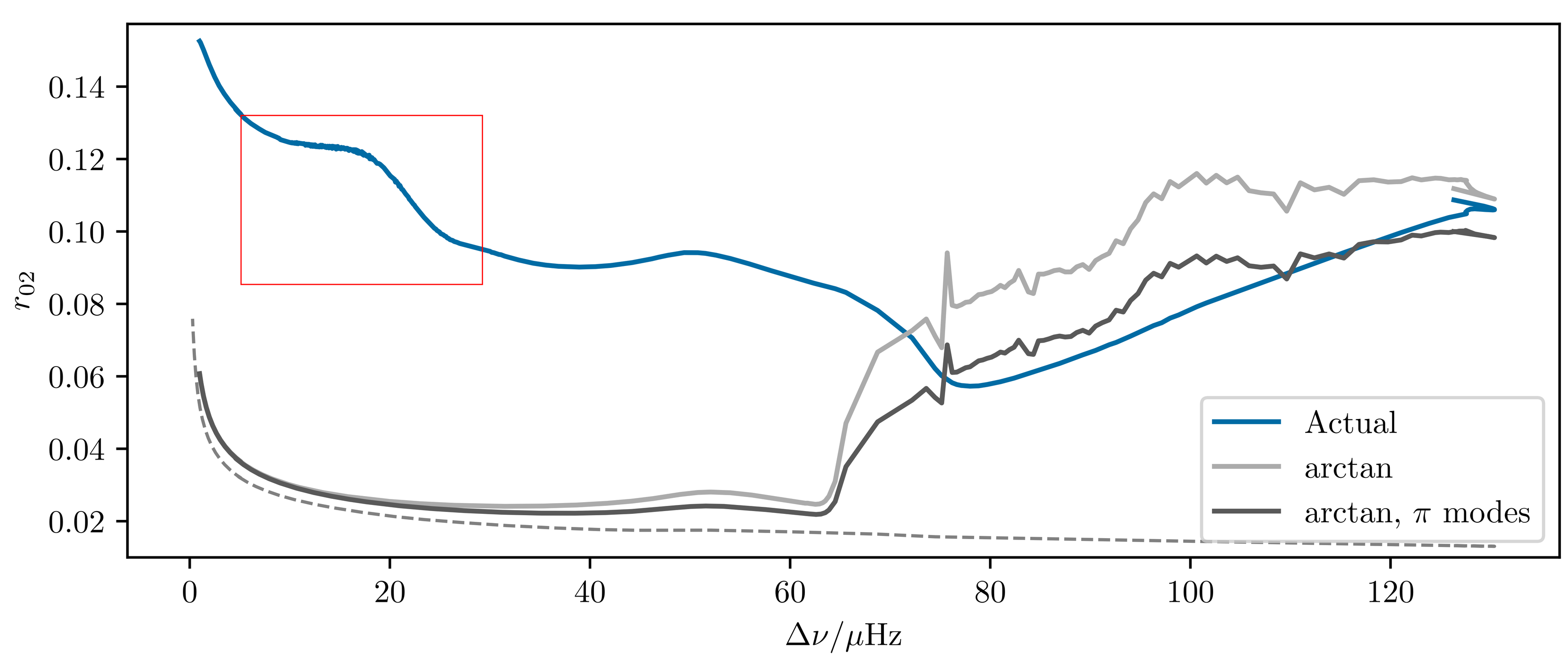

Figure 2: Comparison of revised asymptotic expressions (gray, with dark gray showing further modification to accommodate π modes per Ong & Basu 2020) against numerical results from the same evolutionary track as fig. 1. Dashed line shows limiting behaviour as $A_0 \gg 2\pi\nu_{\max}$. The featured marked in red corresponds to the kink in the C-D diagram that was recently discovered observationally (see Claudia Reyes' poster).

Our full revised asymptotic expression for r_{02} reproduces at least the qualitative behaviour of numerical mode frequencies. **Residual deviations from it** are the result of higher-order contributions from the Brunt-Väisälä frequency, such as arising from acoustic glitches, and **remain indicative of internal structural features**.

Interpreting Observed C-D Diagrams

We apply this analysis to explain a newly-observed kink in the C-D diagram of M67 — **see poster of Claudia Reyes**. Phase-function analysis of numerical eigenfunctions indicates that this kink is caused by a structural feature near the centre of the star, affecting only the frequencies of radial modes.

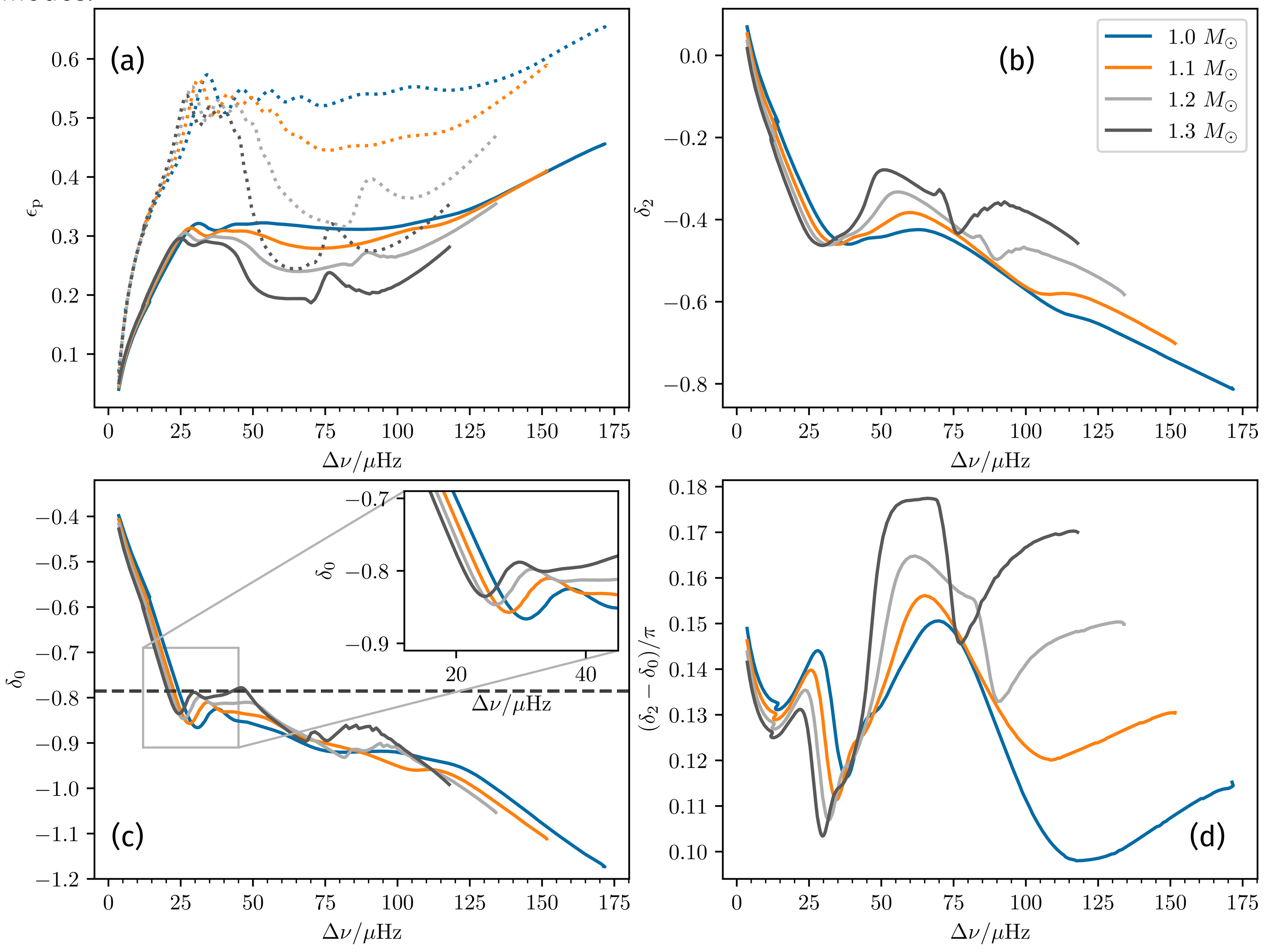

Figure 3: Evolution of several phase functions for different stellar masses. The kink in r_{02} can be seen to originate from only the radial-mode phase function δ_0, and so also affects $\epsilon_p = \frac{1}{\pi}(\delta_0 - \alpha)$. δ_2 evolves smoothly there.

The frequencies of radial modes in particular are fully described by an acoustic potential function V_0. **Departures from our asymptotic expression** are described by its perturbation kernels (fig. 4a).

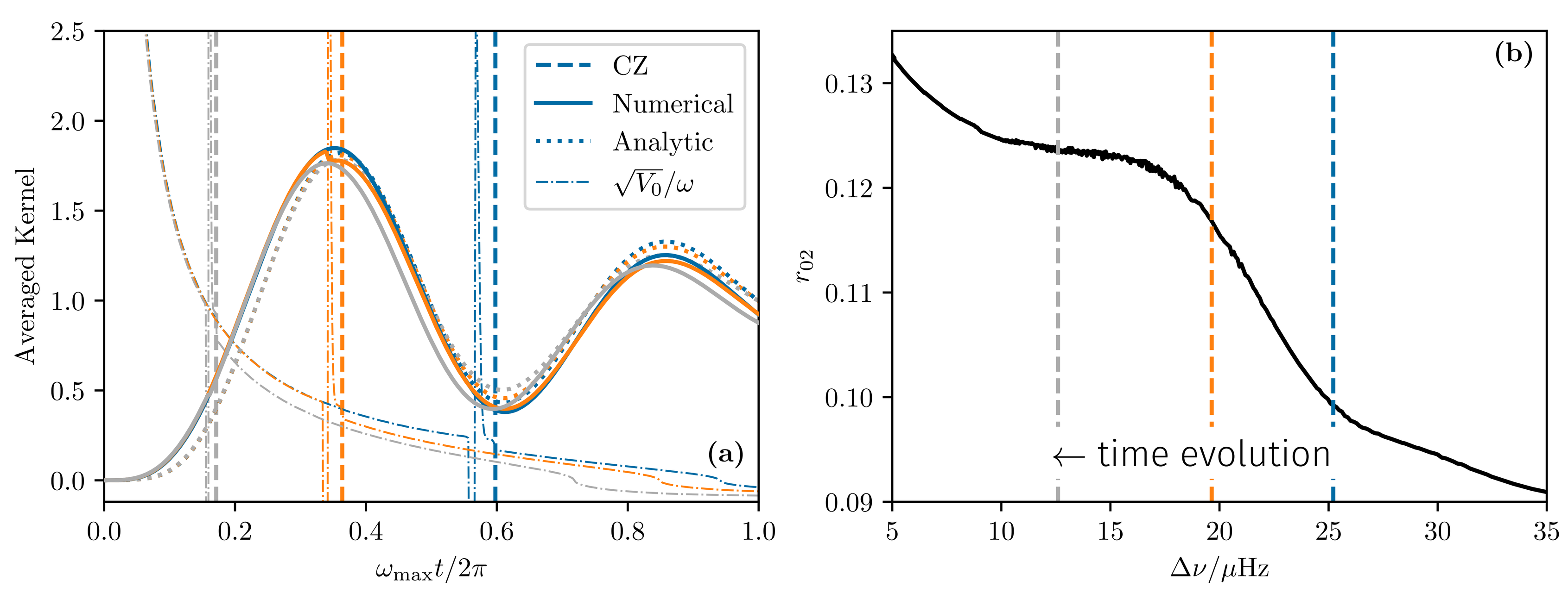

Figure 4: Position of convective-envelope acoustic glitch, in the acoustic potential V_0, relative to averaged perturbation kernel near $\nu_{\max}$. Each set of curves and lines in the left panel is plotted using the same colour as the line denoting position on the C-D diagram in the right panel.

Acoustic glitches in V_0 exist at convective boundaries. Specifically, at the base of the envelope,

- The C-D kink, fig. 4b, emerges when the glitch **sweeps over the innermost local maximum** of the averaged kernel, fig. 4a.
- The shape of this averaged kernel is **extremely stable** over the course of stellar evolution, with respect to the **acoustic phase coordinate** $2\pi\nu_{\max}t$.

Using δ_0 from earlier, the C-D diagram kink arises when convective envelope boundary reaches $t_{\text{glitch}} \sim 0.35/\nu_{\max}$, over a large range of possible stellar masses and ages!

- This kink is a **sensitive observational diagnostic** of envelope overshoot; we have now related it to **internal structure**.

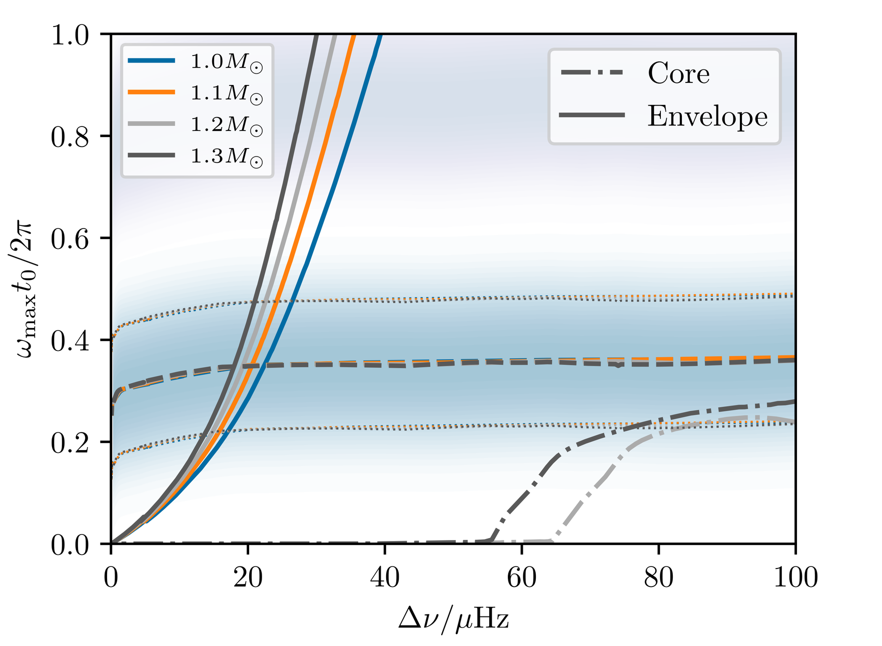

Figure 5: Evolution of the kernel functions, and the locations of the convective boundary glitches, as a function of the acoustic phase radial coordinate $2\pi\nu_{\max}t$.

References

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