# Interactions between differential rotation and mixed-mode coupling in evolved stars

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Existing techniques for rotational characterisation of evolved stars operate well only **within a narrow range** of post-main-sequence evolution. We generalise these techniques beyond the **two-zone model** of radial differential rotation.

## Motivation

Conventional treatments of the interaction between mode mixing and differential rotation:

$$\delta\omega_{\mathrm{rot}} \sim m\left(\zeta\beta_g\Omega_{\mathrm{core}} + (1-\zeta)\beta_p\Omega_{\mathrm{env}}\right). \quad (1)$$

Actual structure of rotational splittings is more complicated (fig. 1).

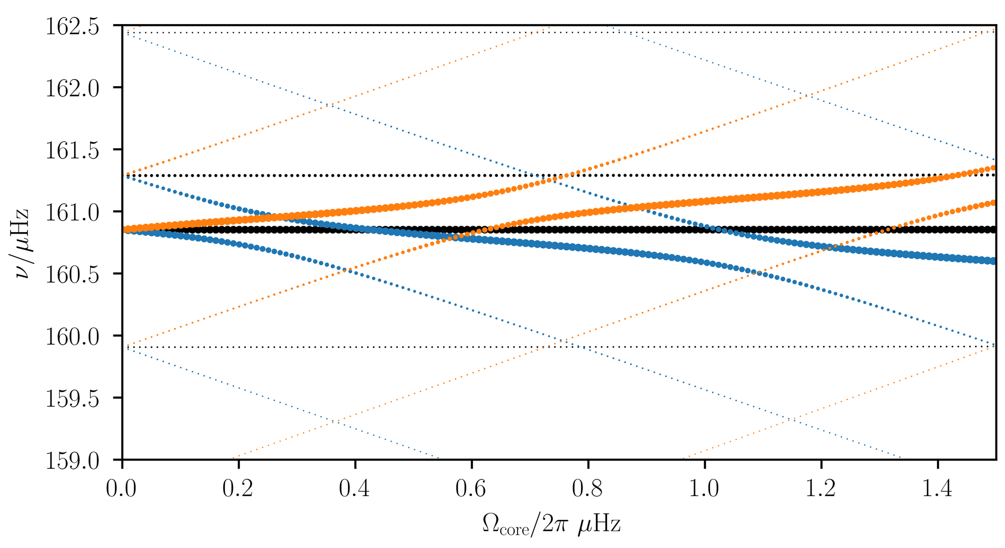


**Figure 1:** Avoided crossings between modes of same $m$ result from differential rotation near p/g-mode resonances

- $\zeta$-function/stretched-échelle approach **not applicable** outside asymptotic regime; does not work well near **p-dominated mixed modes**; does not treat **envelope rotation** (fig. 2).

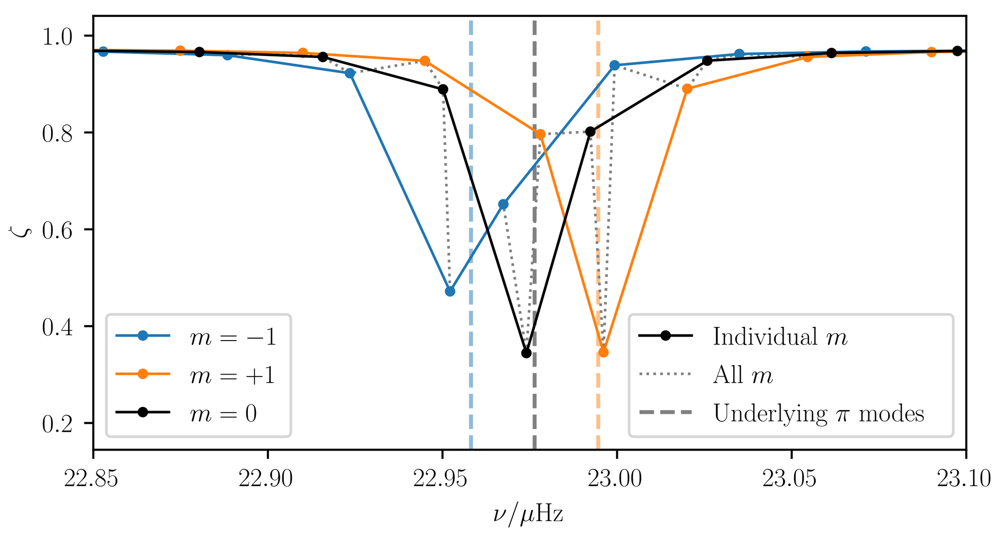


**Figure 2:** In presence of envelope rotation, modes of different $m$ satisfy different mixing functions $\zeta$, and cannot be straightened together on the same stretched échelle diagram.

- Equation (1) **not generalisable** to many-to-one mode coupling, or outside **two-zone model** of differential rotation.

**Under what conditions are existing studies of differential rotation with mixed modes reliable? How may we go beyond the two-zone model?**

## How well do asymptotic methods work?

Hare-and-hounds exercise with MESA models and angular momentum transport: estimates for various kinds of **systematic errors** incurred by asymptotic stretched-echelle procedure. (figs. 3 and 4)

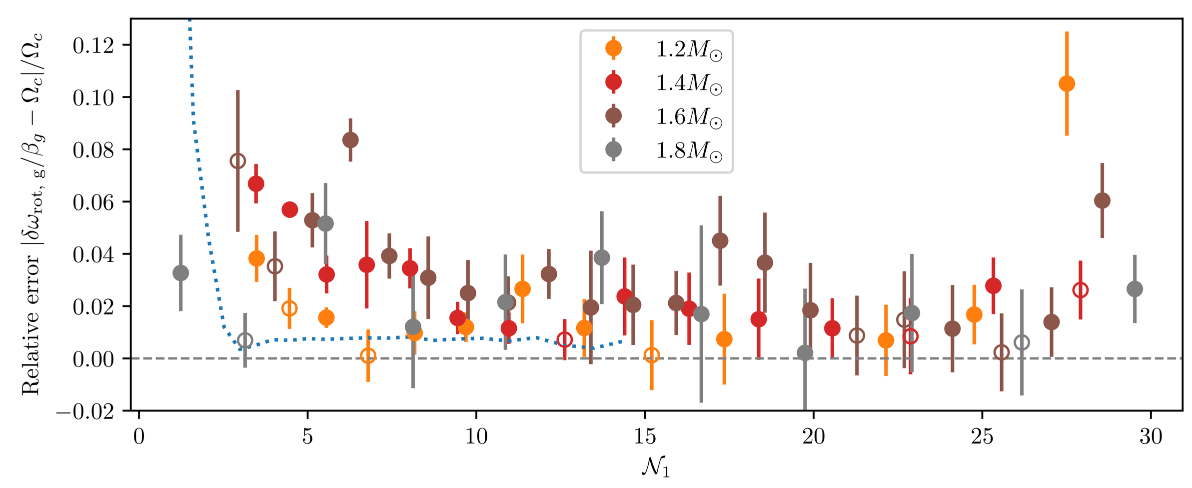


**Figure 3:** Relative systematic errors incurred from stretched-echelle diagrams. Blue line indicates intrinsic error from JWKB approximation. Open circles denote negative values.

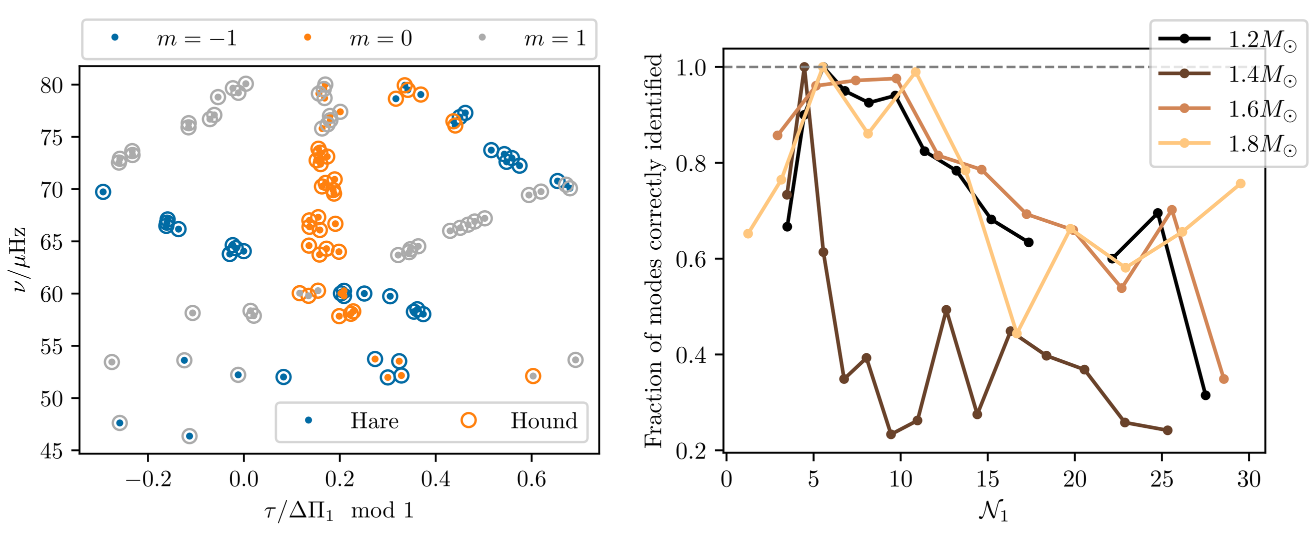


**Figure 4:** Multiplet misidentification in stretched-echelle procedure. Left: hare vs. hound multiplet component identification showing significant misidentification. Right: Misidentification worsens with evolution.

## Mode Coupling and Rotation Matrices

Rotation interacts with mode coupling matrices via a **Quadratic Hermitian Eigenvalue Problem** (Lynden-Bell & Ostriker, 1967):

$$\left(\omega^2\begin{bmatrix}\mathbb{I}_p & \mathbf{D}\\ \mathbf{D}^\dagger & \mathbb{I}_g\end{bmatrix} + \omega\mathbf{R}(m) + \begin{bmatrix}\boldsymbol{\Omega}_p^2 & \mathbf{A}\\ \mathbf{A}^\dagger & \boldsymbol{\Omega}_g^2\end{bmatrix}\right)\mathbf{x} = 0. \quad (2)$$

- Expressions like eq. (1) neglect **off-diagonal elements** of rotation matrix **R**.
- Poor approximation in mixed-mode basis, but is good in **isolated basis of $\pi/\gamma$ modes** (fig. 5).

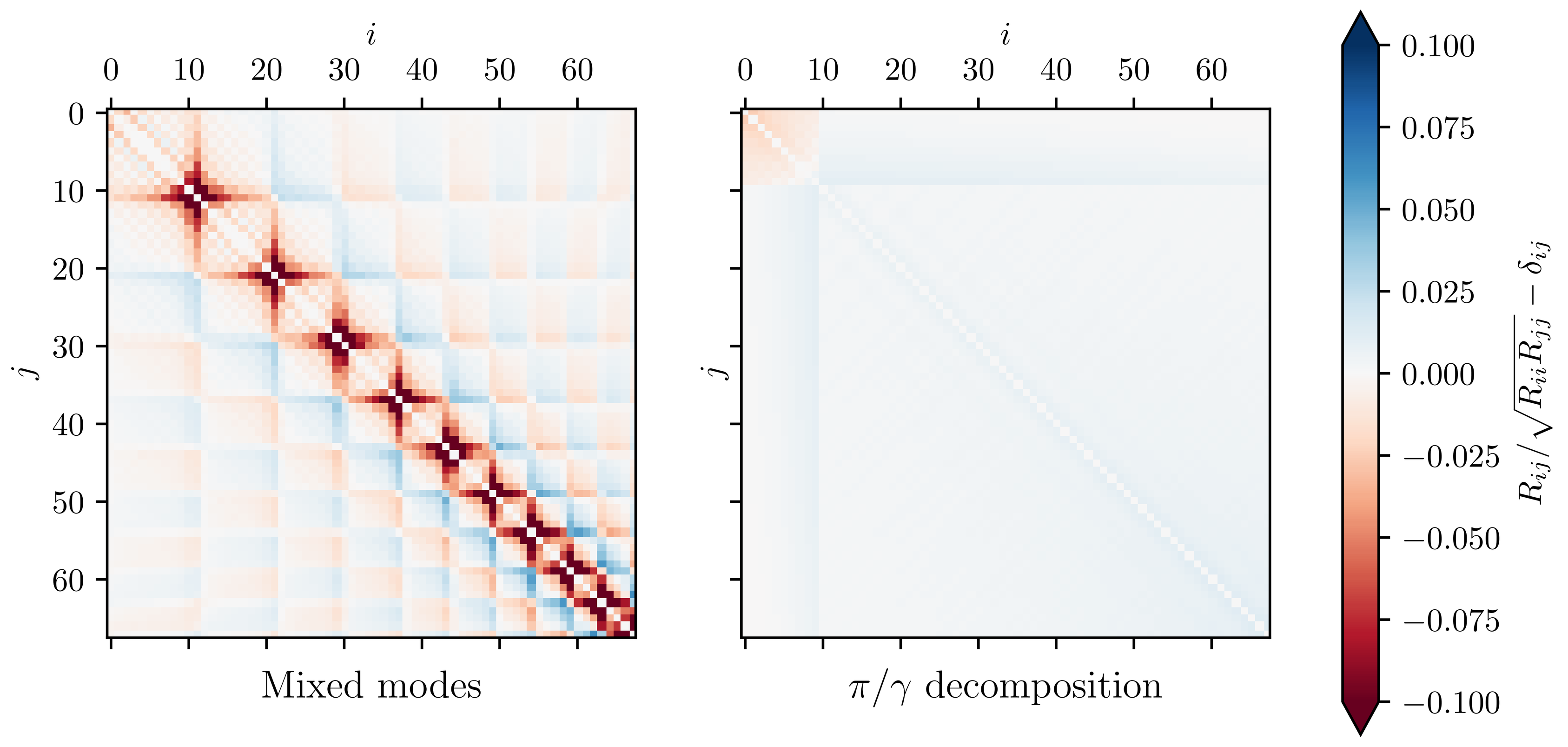


**Figure 5:** Scaled off-diagonal elements of rotation matrix **R** in mixed-mode basis (left) and in $\pi/\gamma$-mode basis (right)

$\implies$ $\pi$ and $\gamma$ modes provide natural basis for symmetrisation of rotational splittings!

## Isolated Inversion Kernels

Rotational inversion kernels rely on eq. (1), and are sensitive to variations in mixing fractions $\zeta$.

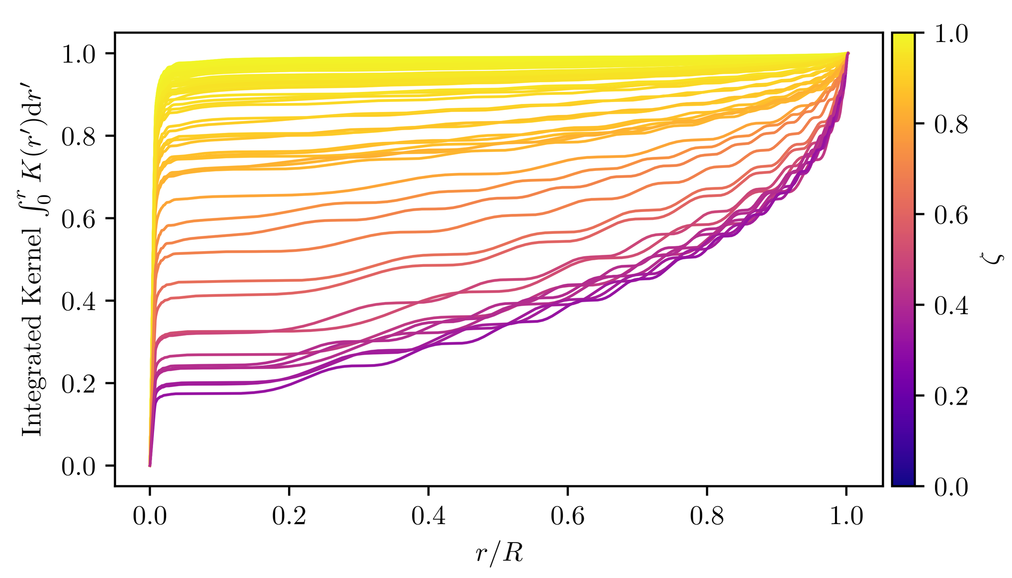


**Figure 6:** Integrated mixed-mode sensitivity kernels, showing strong dependence on mixing fraction

By contrast, the isolated $\pi$ and $\gamma$ modes yield rotational kernels which are entirely localised to **either the core or the envelope** (fig. 7).

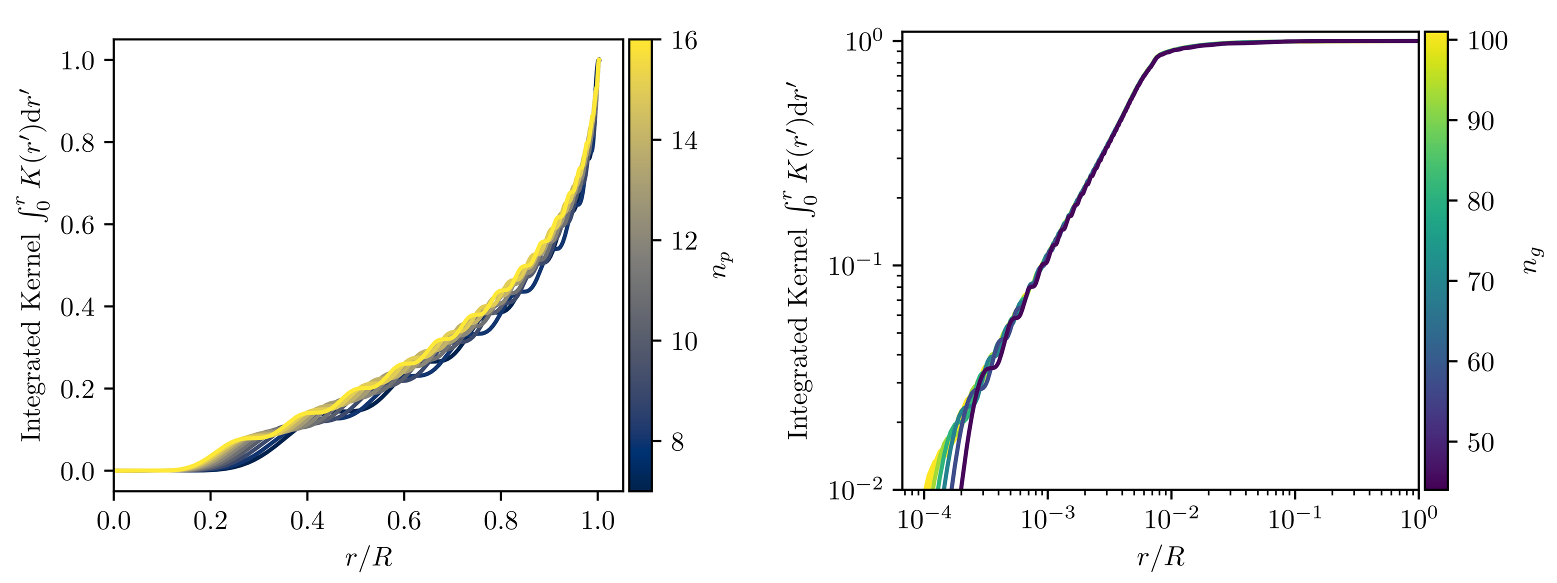


**Figure 7:** Integrated isolated $\pi$- (left) and $\gamma$-mode (right) rotational kernels

$\implies$ Disentangles differential rotation in core/envelope from mode mixing!

## References

Papers forthcoming: Ong, Bugnet, & Basu, submitted to ApJ; Ong & Gehan, in prep.

Lynden-Bell, D., & Ostriker, J. P. 1967, MNRAS, 136, 293