Structural and Evolutionary Diagnostics from Asteroseismic Phase Functions

TASC5/KASC12

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July 22, 2019

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Motivation

Asteroseiseismology in a nutshell

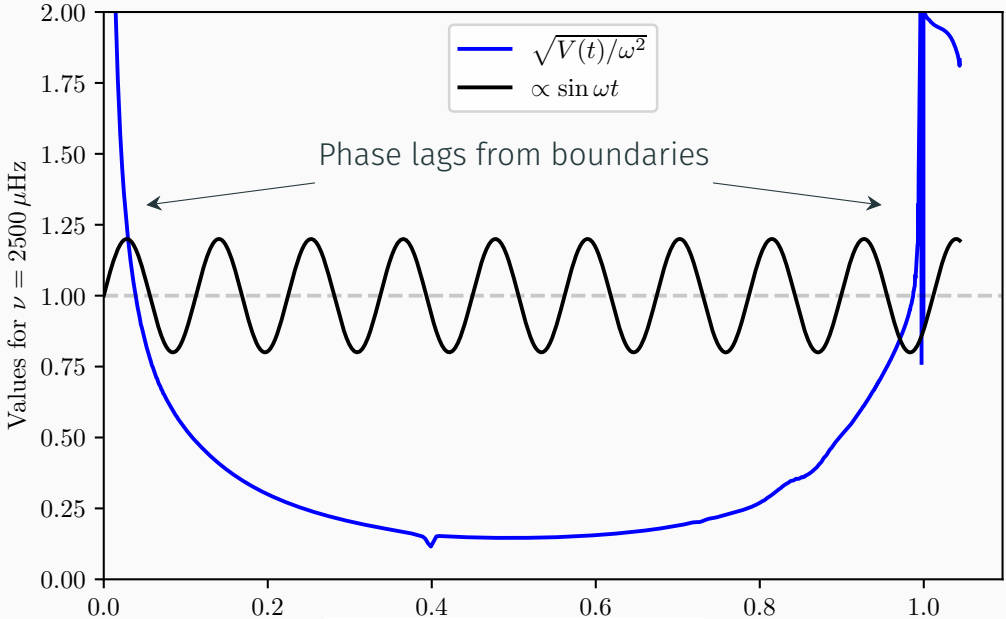
Frequencies of oscillation emerge as eigenvalues for solutions to a boundary-value problem:

Phase offset

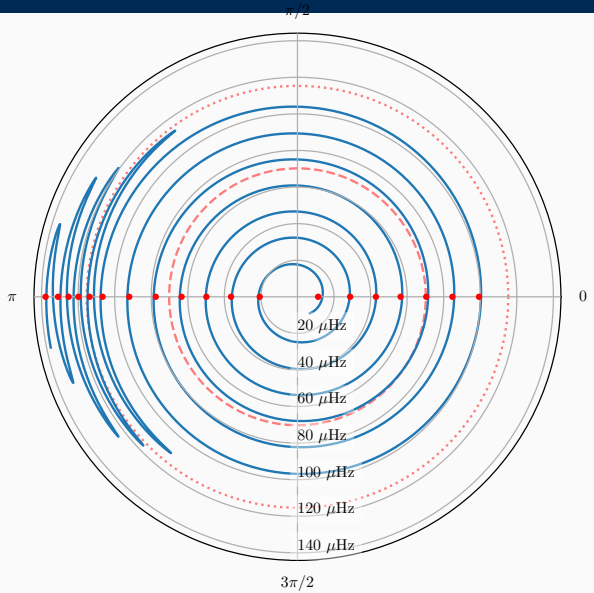
$$\theta(\omega) = \omega T_0 - \pi \underbrace{\epsilon_l(\omega)}_{\text{boundary conditions}} = \overbrace{\pi\left(n + \frac{l}{2}\right)}^{\text{homogeneous spherical well}}$$

$$\Downarrow$$

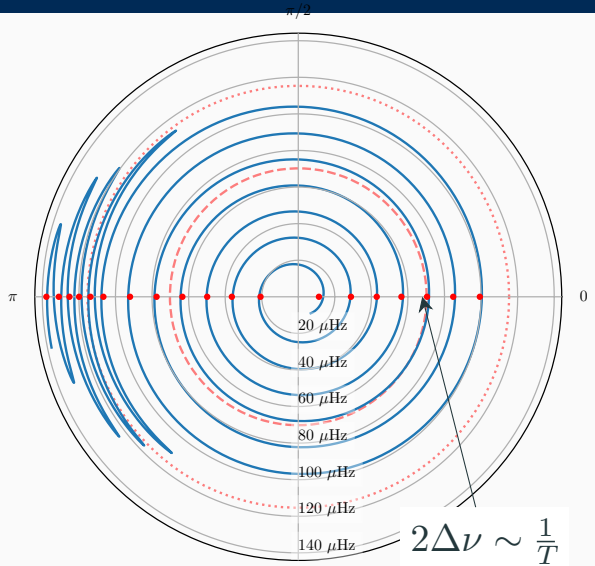
$$\nu_{nl} = \frac{1}{2T_0}\left(n + \frac{l}{2} + \epsilon_l(\nu_{nl})\right)$$


$\sqrt{V(t)/\omega^2}$
$\propto \sin \omega t$
Phase lags from boundaries
Values for $\nu = 2500\,\mu$Hz
Fractional acoustic radius
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
0.0
0.2
0.4
0.6
0.8
1.0

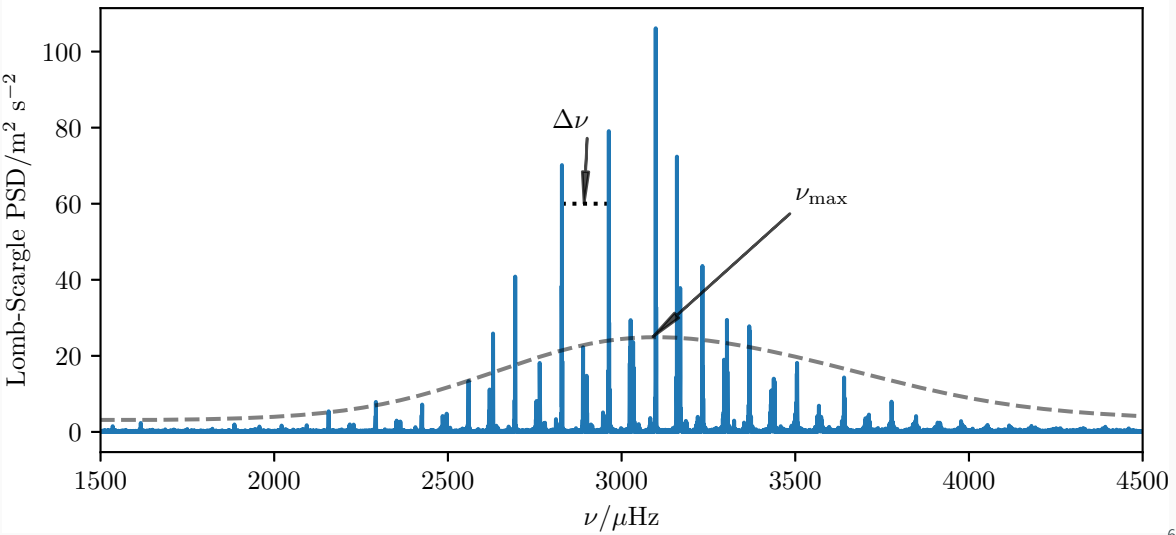
Eigenvalue equation: $\theta(\omega) = \omega T - \pi\epsilon_l(\omega) = \pi\left(n + \frac{l}{2}\right)$



Eigenvalue equation: $\theta(\omega) = \omega T - \pi\epsilon_l(\omega) = \pi\left(n + \frac{l}{2}\right)$



What the observer sees



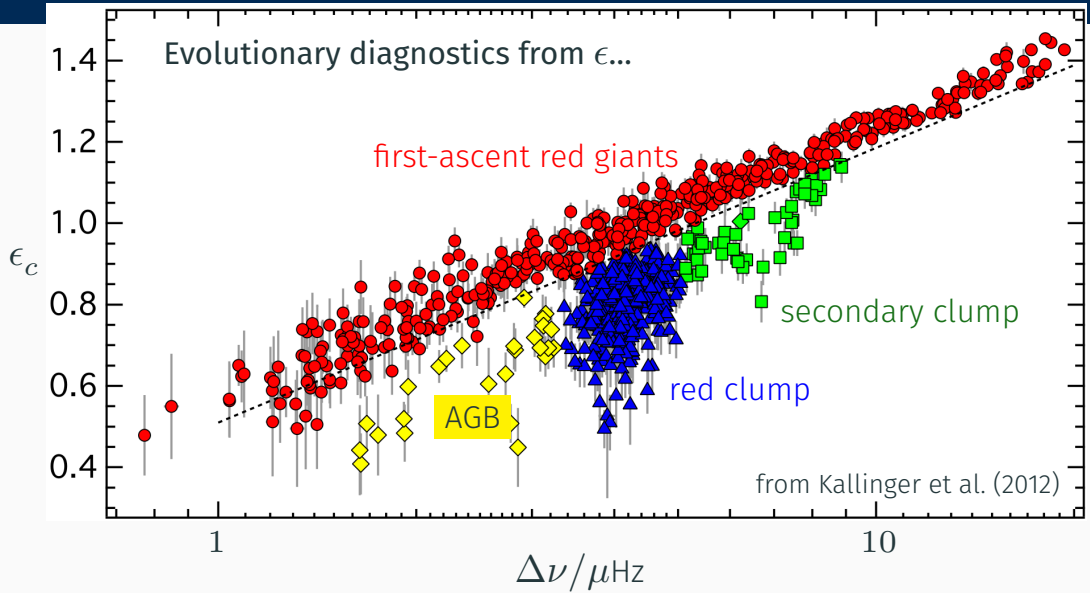
Asteroseismology in a nutshell (“data-driven” version)

- Approximate eigenvalue equation of the form

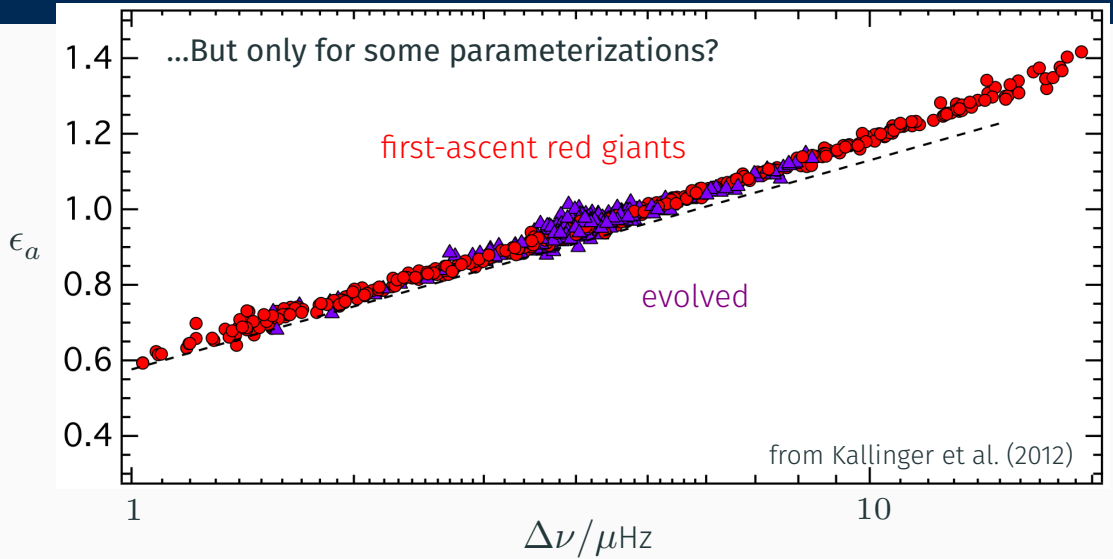
$$\nu_{nlm} \sim \Delta\nu \left(n + \frac{l}{2} + \epsilon_l \right), \; \Delta\nu \sim 1/2T_0.$$

- Scaling relations for $\Delta\nu$ and ν_{max} in terms of other stellar parameters

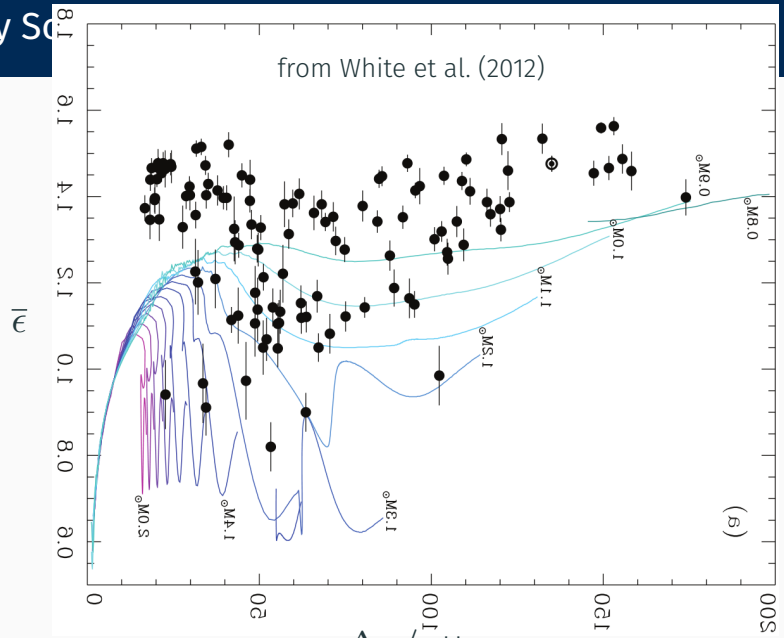
The Story So Far



The Story So Far



The Story So


from White et al. (2012)
ϵ
$\Delta\nu/\mu$Hz
0
50
100
150
200
8.0
8.0
8.1
8.1
8.1
8.1
8.1
(g)
0.8M⊙
0.9M⊙
1.0M⊙
1.1M⊙
1.3M⊙
1.4M⊙
1.5M⊙

Estimators of ϵ

$\overline{\epsilon}$ (“intercept”)

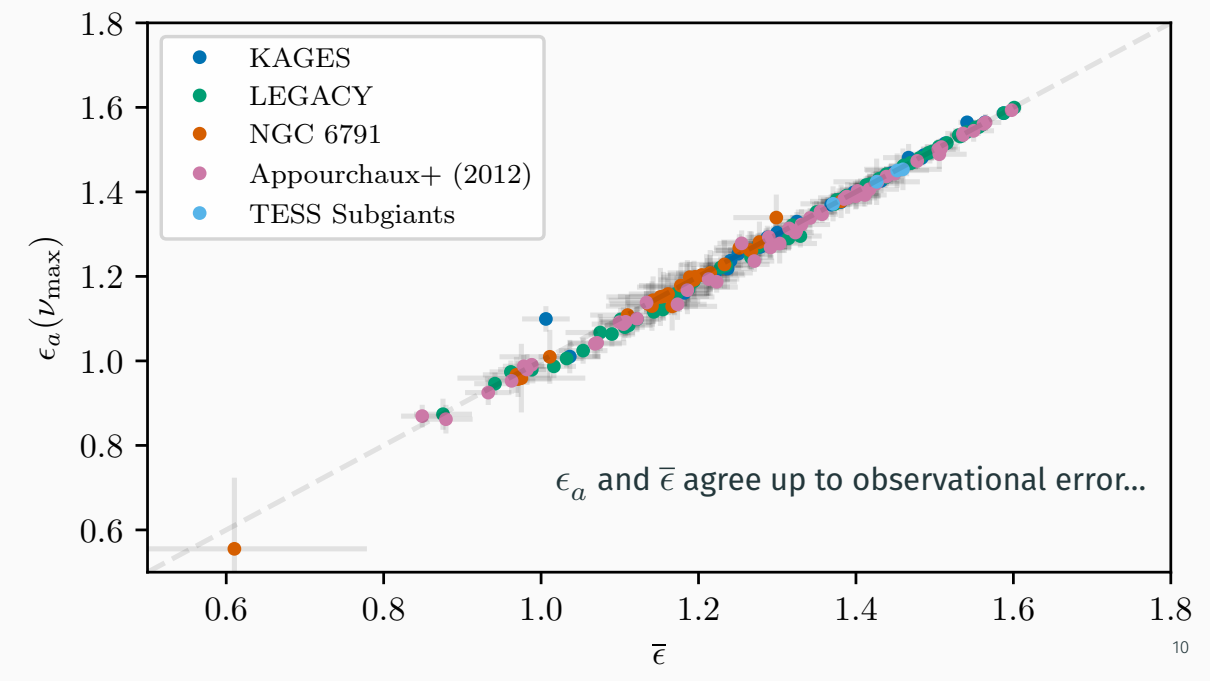
y-intercept to linear fit (ν vs n) that also returns $\Delta\nu$

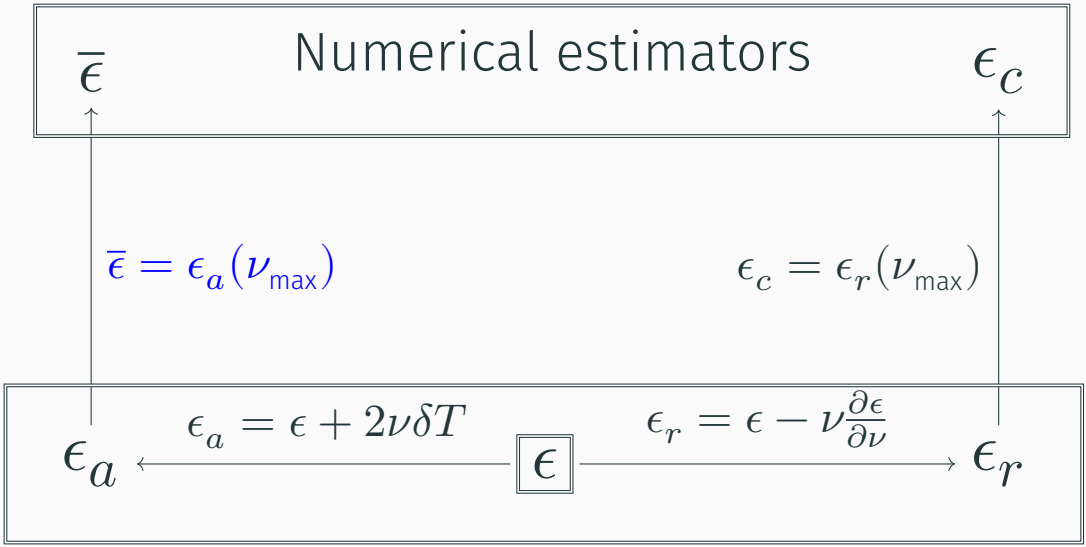
ϵ_a (“average”)

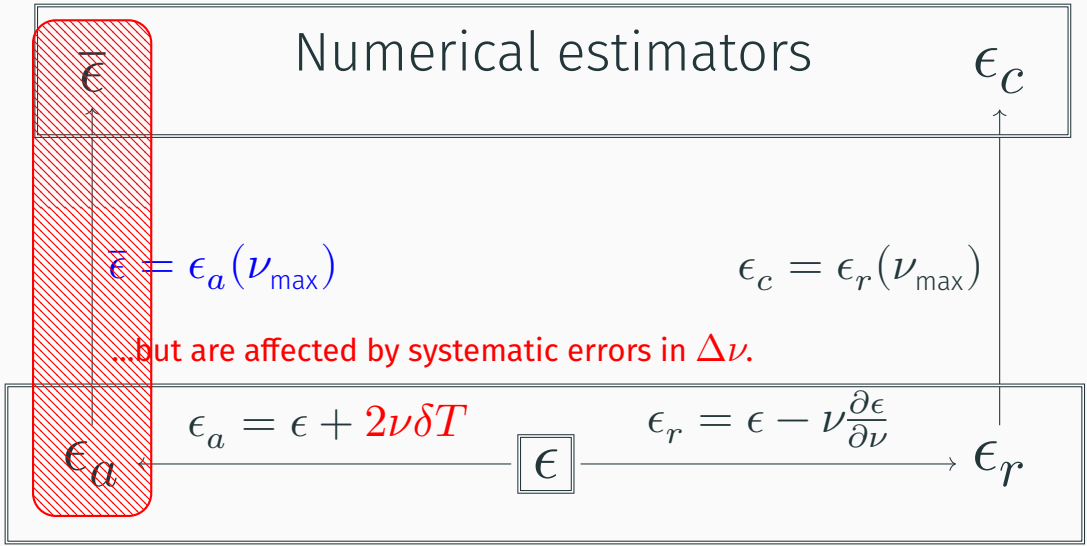
$$\epsilon_a \sim \left\langle \frac{\nu}{\Delta\nu} - n \right\rangle$$

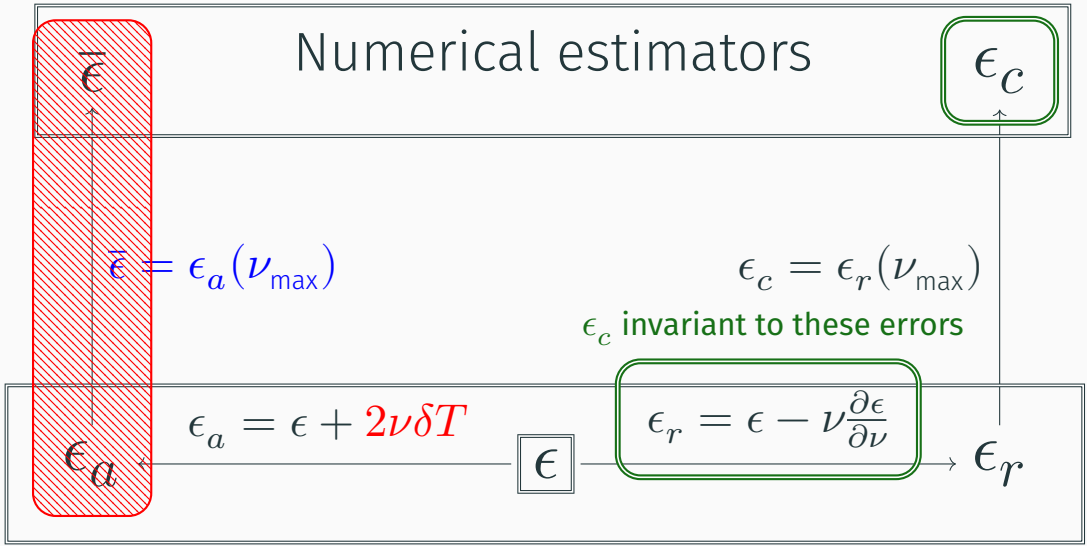
ϵ_c (“central”)

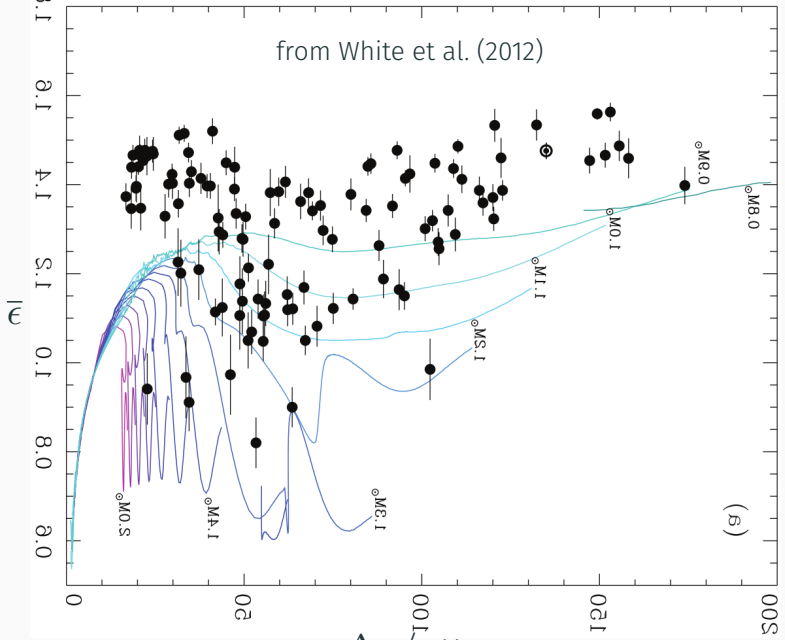
$$\epsilon_c \sim \frac{\nu_c}{\Delta\nu_c} - n_c$$

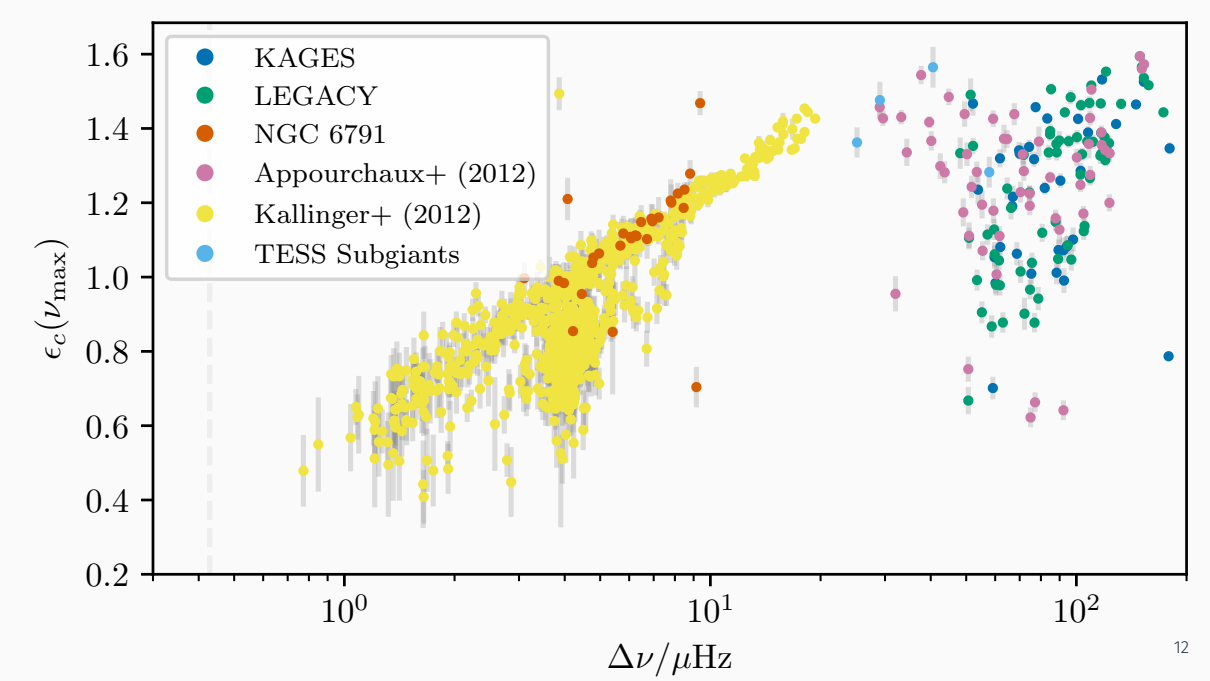

KAGES
LEGACY
NGC 6791
Appourchaux+ (2012)
TESS Subgiants
ϵ_a and $\overline{\epsilon}$ agree up to observational error...
$\epsilon_a(\nu_{\max})$
$\overline{\epsilon}$
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.6
0.8
1.0
1.2
1.4
1.6
1.8

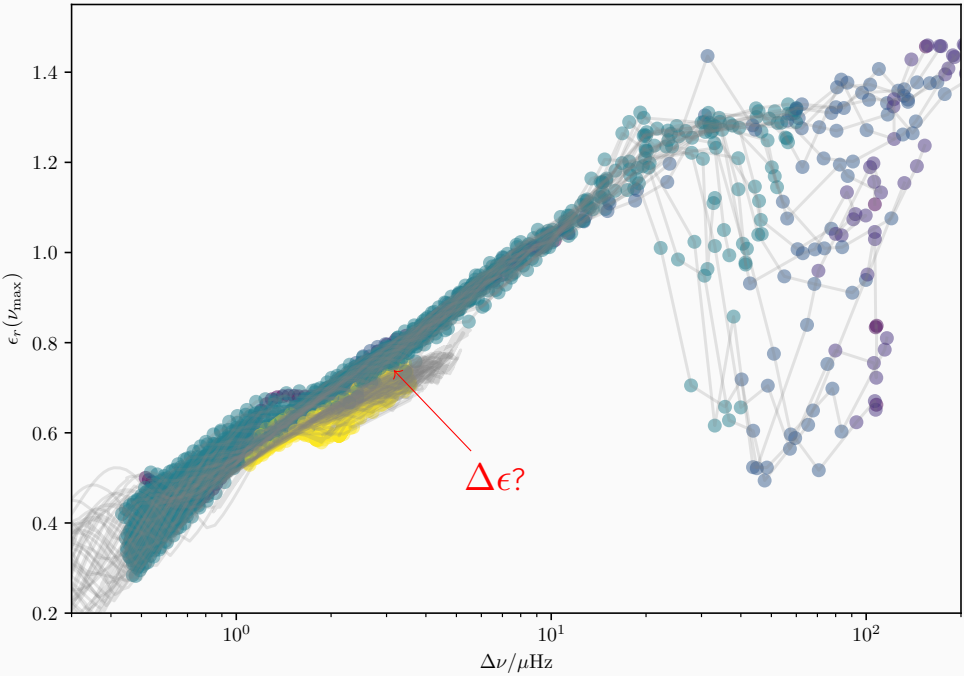

Numerical estimators
$\overline{\epsilon}$
ϵ_c
$\overline{\epsilon} = \epsilon_a(\nu_{\max})$
$\epsilon_c = \epsilon_r(\nu_{\max})$
ϵ_a
$\epsilon_a = \epsilon + 2\nu\delta T$
ϵ
$\epsilon_r = \epsilon - \nu\frac{\partial\epsilon}{\partial\nu}$
ϵ_r
Functions of frequency

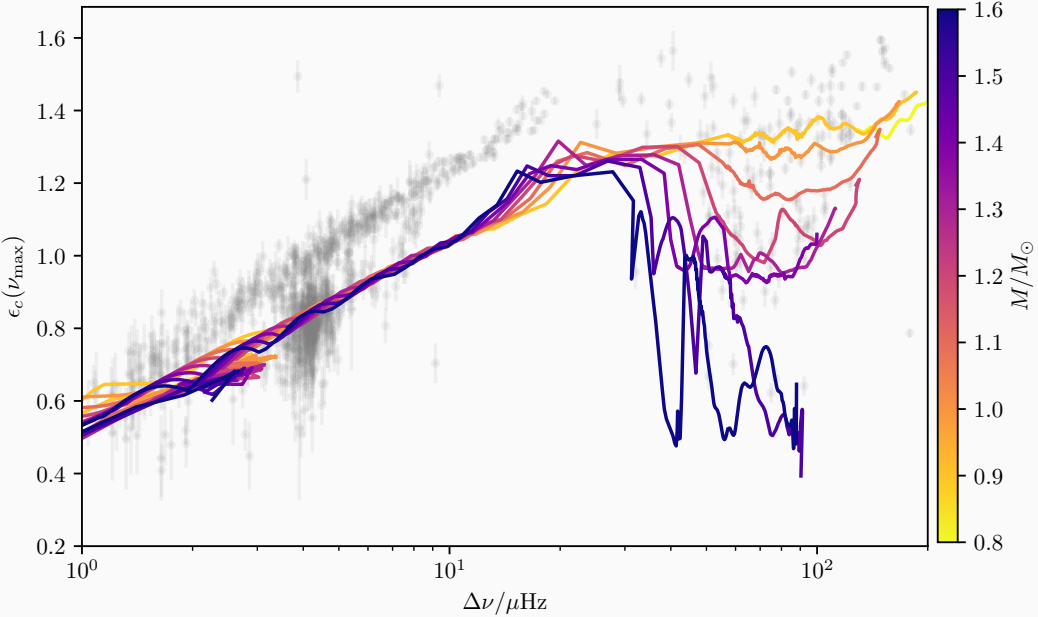

Numerical estimators
ϵ_c
$\bar{\epsilon}$
$\bar{\epsilon} = \epsilon_a(\nu_{\max})$
$\epsilon_c = \epsilon_r(\nu_{\max})$
...but are affected by systematic errors in $\Delta\nu$.
$\epsilon_a = \epsilon + 2\nu\delta T$
$\epsilon_r = \epsilon - \nu\frac{\partial\epsilon}{\partial\nu}$
ϵ_a
ϵ
ϵ_r
Functions of frequency


Numerical estimators
ϵ_c
$\bar{\epsilon}$
$\bar{\epsilon} = \epsilon_a(\nu_{\max})$
$\epsilon_c = \epsilon_r(\nu_{\max})$
ϵ_c invariant to these errors
$\epsilon_a = \epsilon + 2\nu\delta T$
$\epsilon_r = \epsilon - \nu\frac{\partial \epsilon}{\partial \nu}$
ϵ_a
ϵ
ϵ_r
Functions of frequency

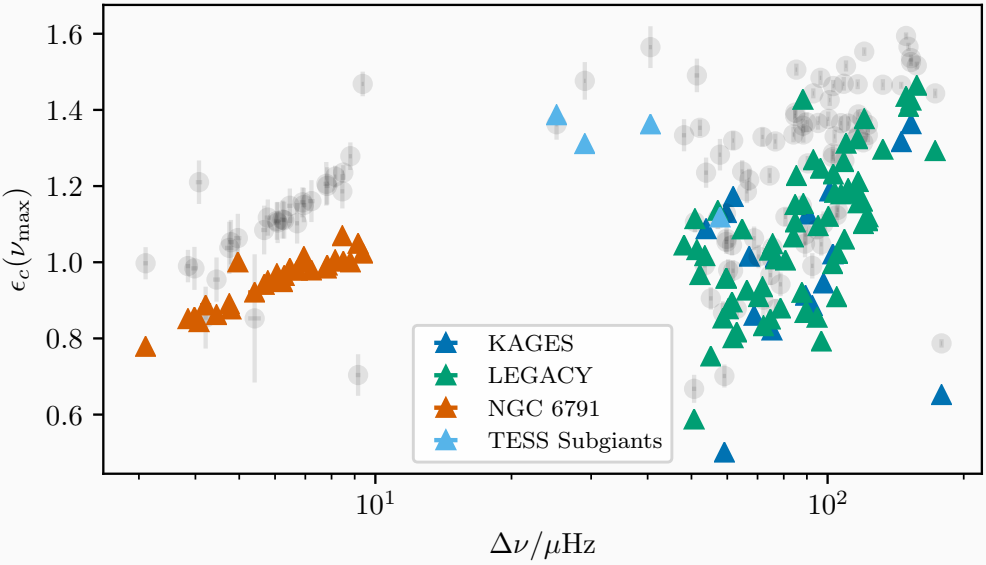

from White et al. (2012)
ϵ
$\Delta\nu/\mu$Hz
0.0
0.8
1.6
2.4
3.2
0
200
400
600
800
1000
1200
1400
0.8 $M_\odot$
1.0 $M_\odot$
1.1 $M_\odot$
1.3 $M_\odot$
1.4 $M_\odot$
2.0 $M_\odot$
(g)


KAGES
LEGACY
NGC 6791
Appourchaux+ (2012)
Kallinger+ (2012)
TESS Subgiants
$\epsilon_c(\nu_{\rm max})$
$\Delta\nu/\mu$Hz
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
10^0
10^1
10^2

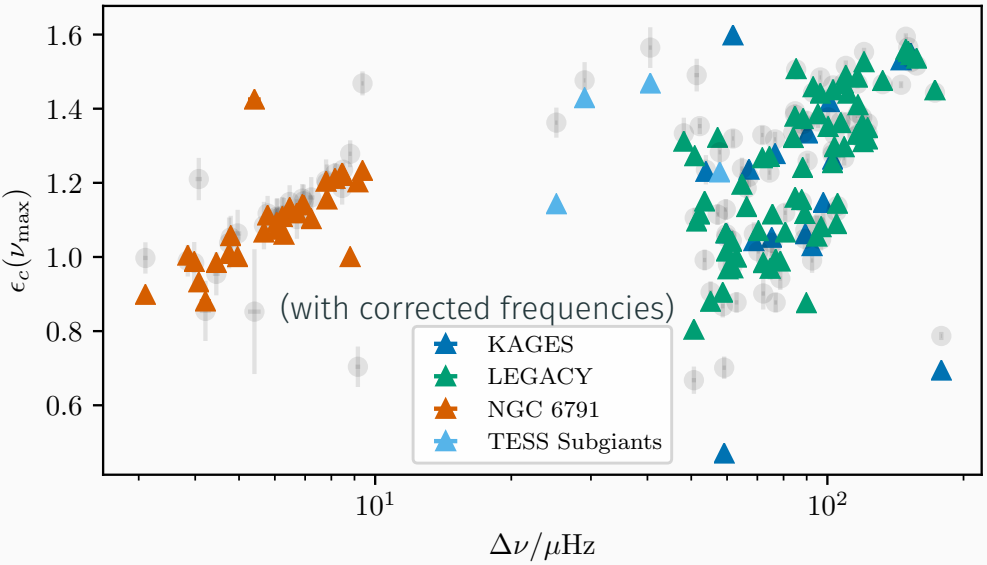

1.4
1.2
1.0
0.8
0.6
0.4
0.2
$\epsilon_r(\nu_{\rm max})$
10^0
10^1
10^2
$\Delta\nu/\mu$Hz
$\Delta\epsilon$?


1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
$\epsilon_c(\nu_{\max})$
10^0
10^1
10^2
$\Delta\nu/\mu$Hz
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
$M/M_\odot$

vs. Best-fitting Models

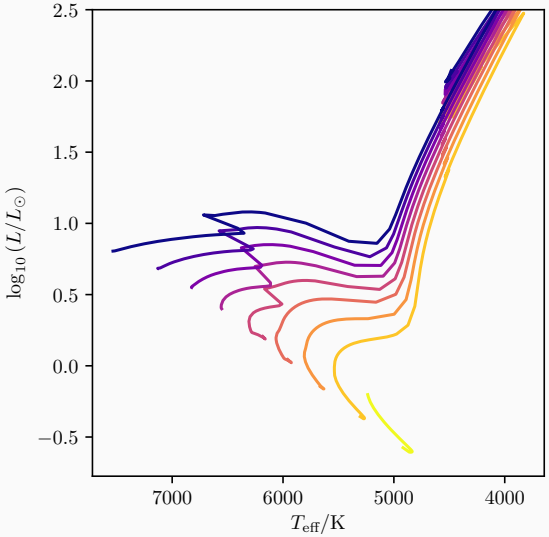


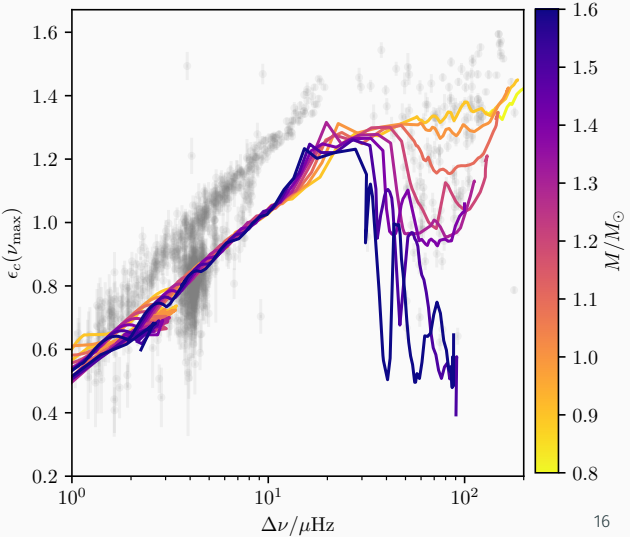
vs. Best-fitting Models



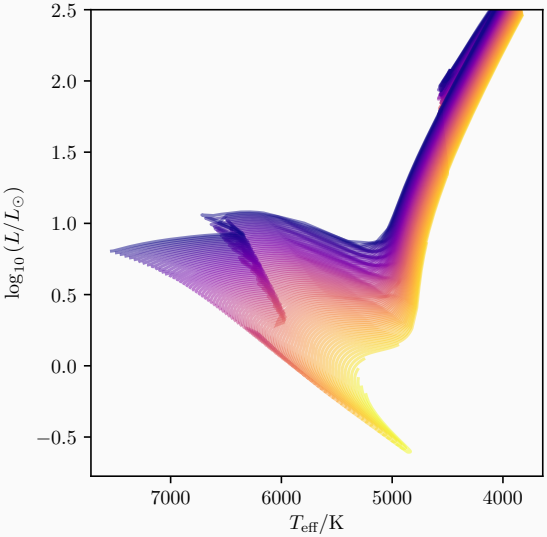
Evolutionary Diagnostics

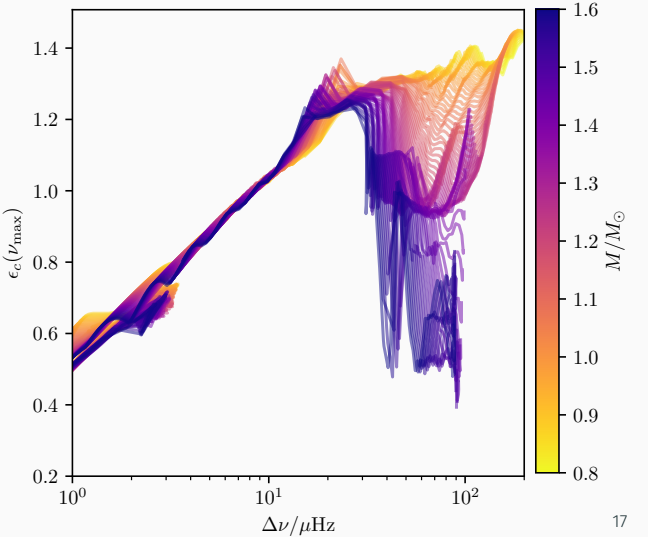
Tracks


2.5
2.0
1.5
1.0
0.5
0.0
−0.5
$\log_{10}(L/L_\odot)$
7000
6000
5000
4000
$T_{\rm eff}$/K

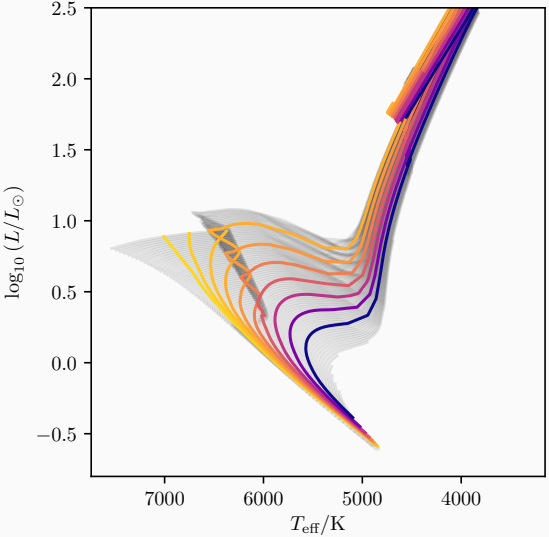

1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
$\epsilon_c(\nu_{\rm max})$
10^0
10^1
10^2
$\Delta\nu/\mu$Hz
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
$M/M_\odot$

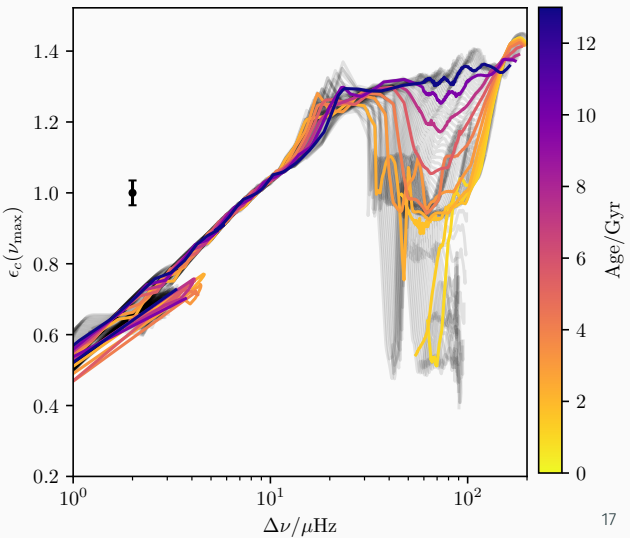
Isochrones


2.5
2.0
1.5
1.0
0.5
0.0
−0.5
$\log_{10}(L/L_\odot)$
7000
6000
5000
4000
$T_{\rm eff}$/K

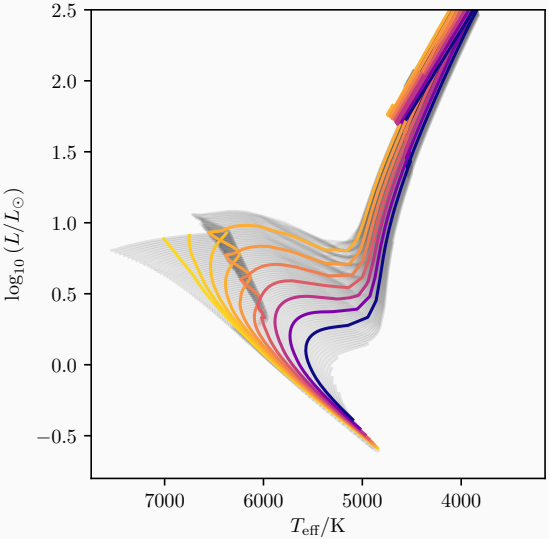

1.4
1.2
1.0
0.8
0.6
0.4
0.2
$\epsilon_c(\nu_{\rm max})$
10^0
10^1
10^2
$\Delta\nu/\mu$Hz
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
$M/M_\odot$

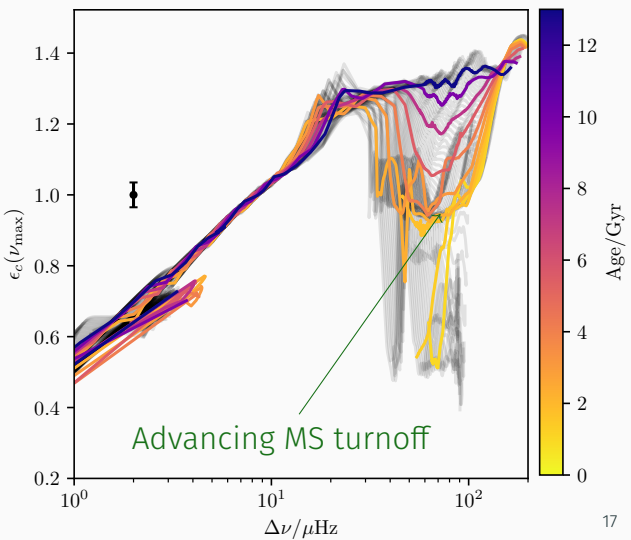
Isochrones


2.5
2.0
1.5
1.0
0.5
0.0
−0.5
$\log_{10}(L/L_\odot)$
7000
6000
5000
4000
$T_{\rm eff}$/K

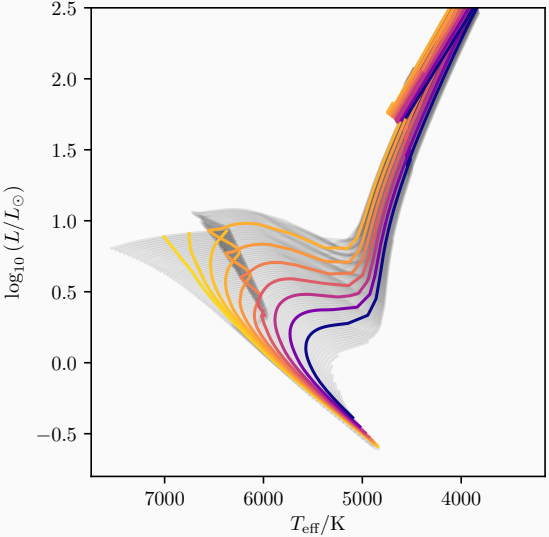

1.4
1.2
1.0
0.8
0.6
0.4
0.2
$\epsilon_c(\nu_{\rm max})$
10^0
10^1
10^2
$\Delta\nu/\mu$Hz
12
10
8
6
4
2
0
Age/Gyr

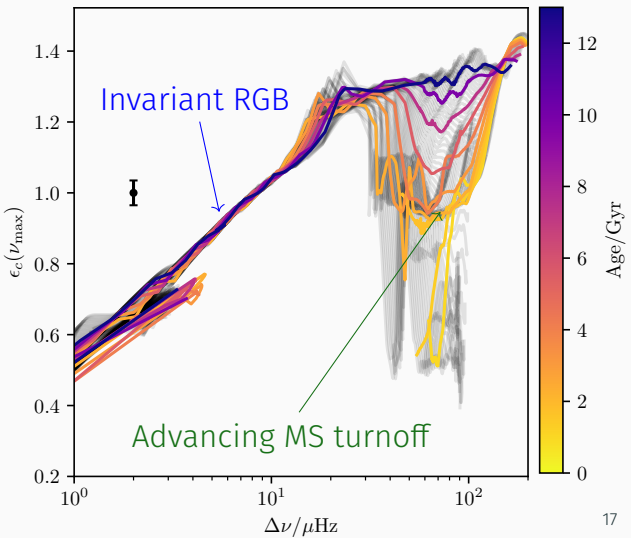
Isochrones


2.5
2.0
1.5
1.0
0.5
0.0
−0.5
$\log_{10}(L/L_\odot)$
7000
6000
5000
4000
$T_{\rm eff}$/K

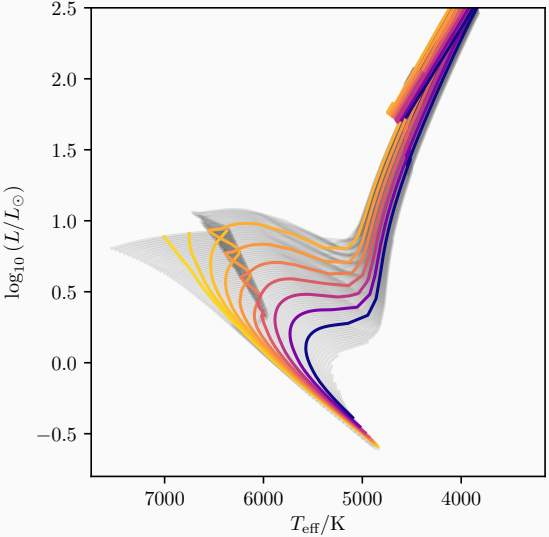

1.4
1.2
1.0
0.8
0.6
0.4
0.2
$\epsilon_c(\nu_{\rm max})$
10^0
10^1
10^2
$\Delta\nu/\mu$Hz
Advancing MS turnoff
12
10
8
6
4
2
0
Age/Gyr

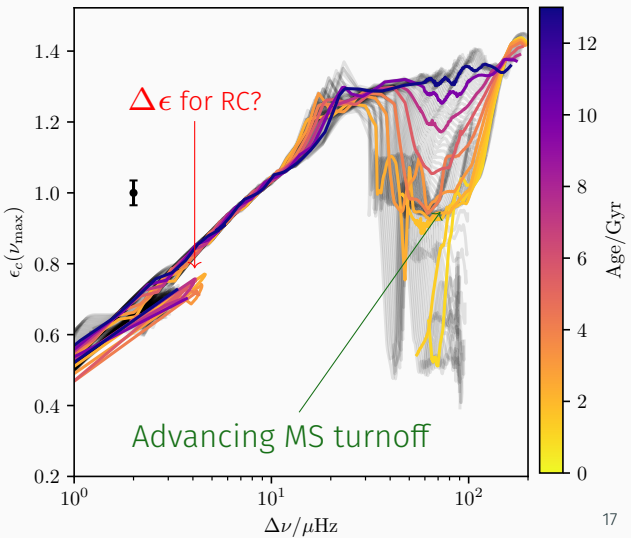
Isochrones


2.5
2.0
1.5
1.0
0.5
0.0
−0.5
$\log_{10}(L/L_\odot)$
7000
6000
5000
4000
$T_{\rm eff}$/K

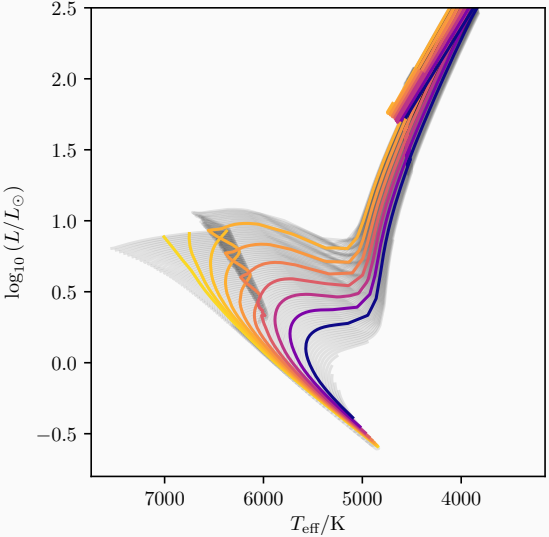

Invariant RGB
Advancing MS turnoff
1.4
1.2
1.0
0.8
0.6
0.4
0.2
$\epsilon_c(\nu_{\rm max})$
10^0
10^1
10^2
$\Delta\nu/\mu$Hz
12
10
8
6
4
2
0
Age/Gyr

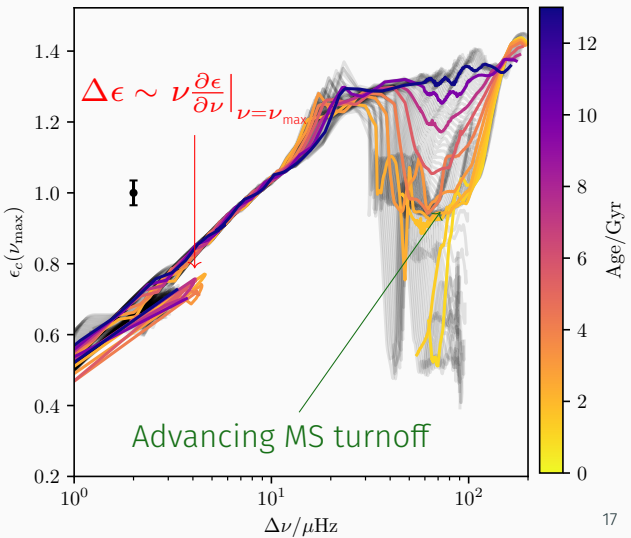
Isochrones


2.5
2.0
1.5
1.0
0.5
0.0
−0.5
$\log_{10}(L/L_\odot)$
7000
6000
5000
4000
$T_{\rm eff}$/K

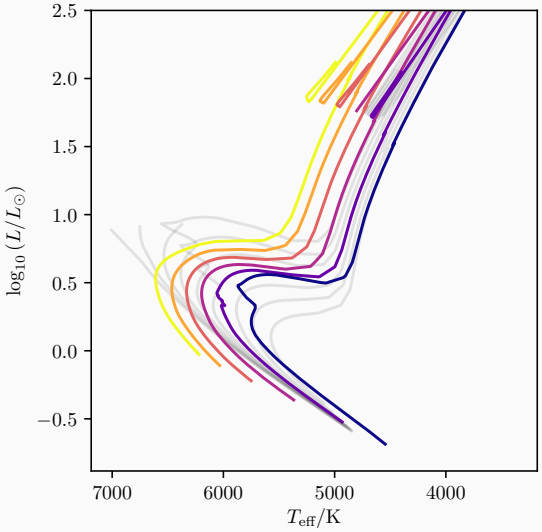

$\Delta\epsilon$ for RC?
Advancing MS turnoff
1.4
1.2
1.0
0.8
0.6
0.4
0.2
$\epsilon_c(\nu_{\rm max})$
10^0
10^1
10^2
$\Delta\nu/\mu$Hz
12
10
8
6
4
2
0
Age/Gyr

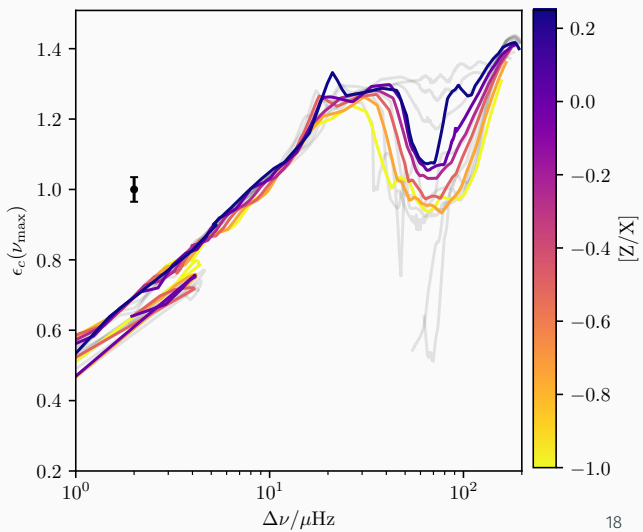
Isochrones


2.5
2.0
1.5
1.0
0.5
0.0
−0.5
$\log_{10}(L/L_\odot)$
7000
6000
5000
4000
$T_{\rm eff}$/K

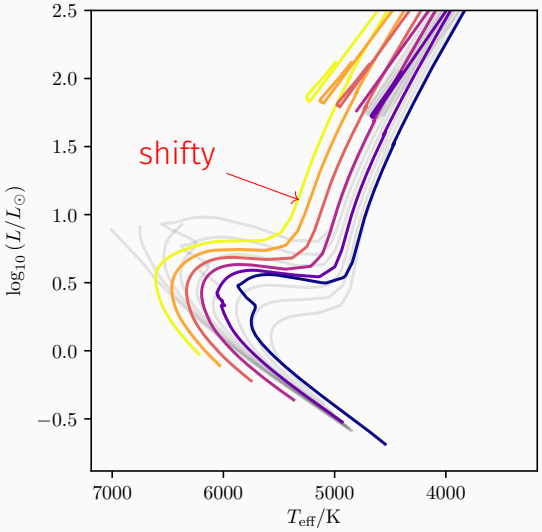

$\Delta\epsilon \sim \nu \frac{\partial \epsilon}{\partial \nu}\Big|_{\nu=\nu_{\rm max}}$
Advancing MS turnoff
1.4
1.2
1.0
0.8
0.6
0.4
0.2
$\epsilon_c(\nu_{\rm max})$
10^0
10^1
10^2
$\Delta\nu/\mu$Hz
12
10
8
6
4
2
0
Age/Gyr

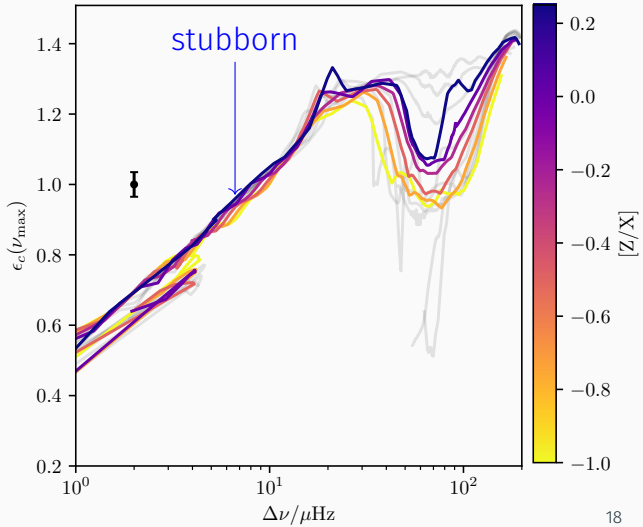
Metallicity



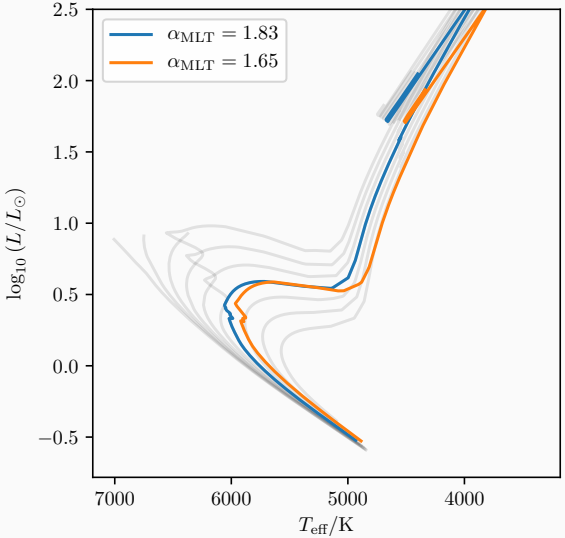


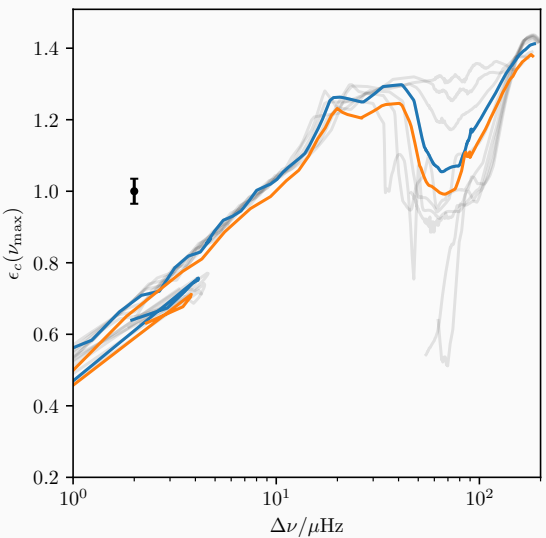
Metallicity



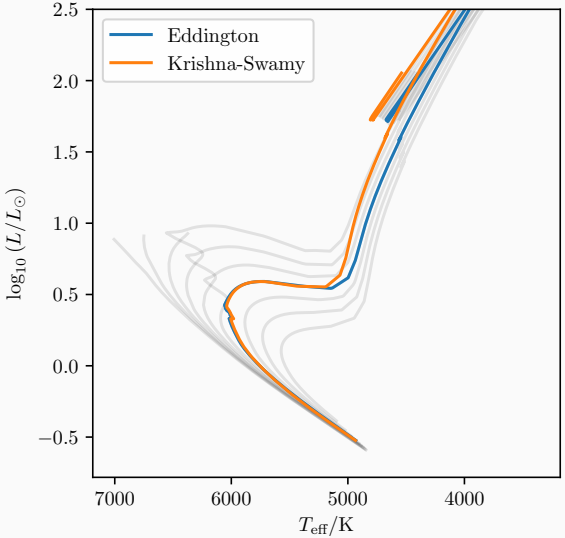


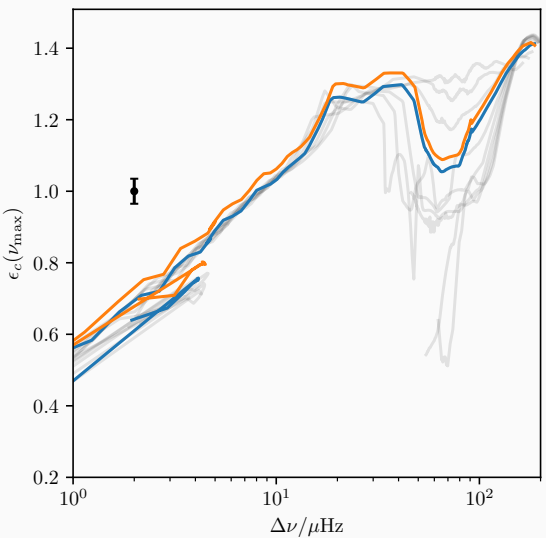
Mixing Length





Atmosphere


Eddington
Krishna-Swamy
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
$\log_{10}(L/L_\odot)$
7000
6000
5000
4000
$T_{\rm eff}$/K


1.4
1.2
1.0
0.8
0.6
0.4
0.2
$\epsilon_c(\nu_{\rm max})$
10^0
10^1
10^2
$\Delta\nu/\mu$Hz

Conclusion

- Phase functions from stellar models sensitive to **evolutionary** and **structural** properties, and **surface** physics.
- $\epsilon_c - \Delta\nu$ diagrams yield isochrones where RGB position is **insensitive to stellar composition** and (some) **modelling choices**
- What happens if you use ϵ_c in a **grid search**?

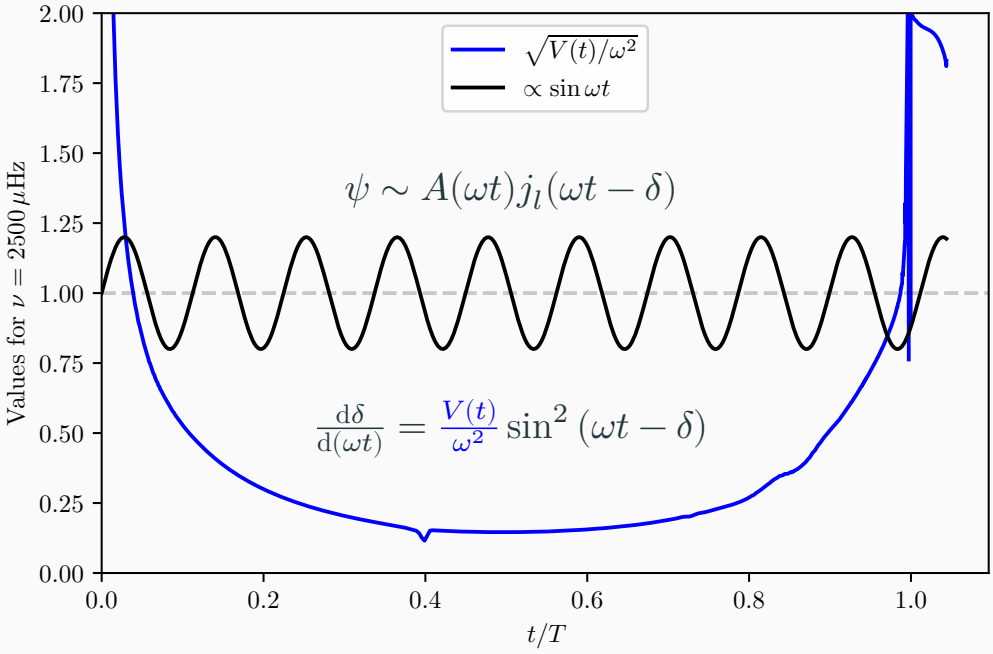
Extra Slides

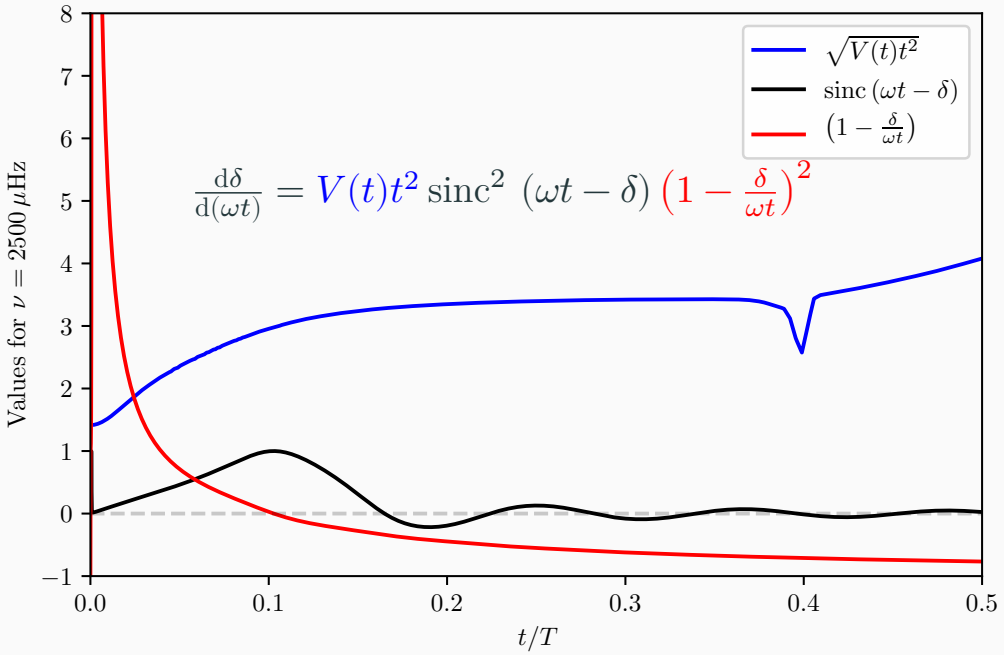
Modelling Parameters

- MESA v10398
  - Diffusion
  - Overshooting
- GYRE v5.2

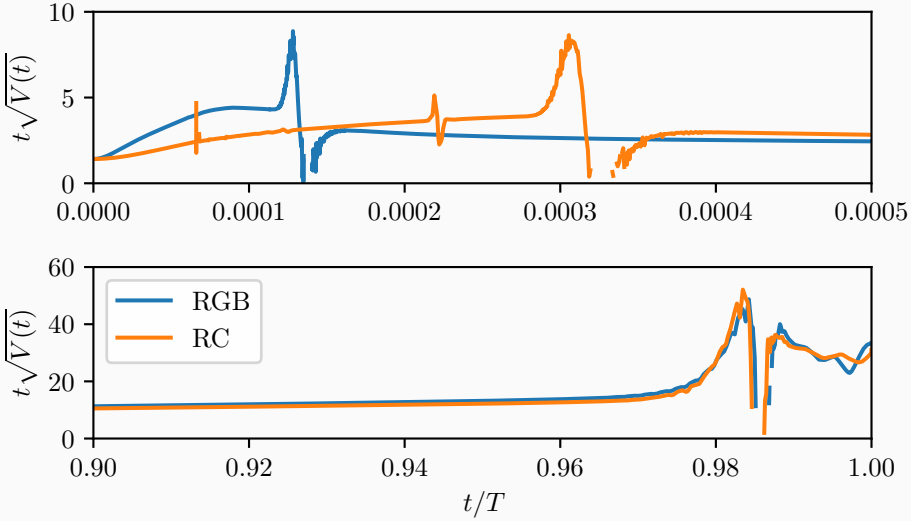
- Solar-calibrated Y_0 and α_{MLT} against Eddington-grey atmospheres and GS98 abundances

(nothing special otherwise — I mostly used defaults)

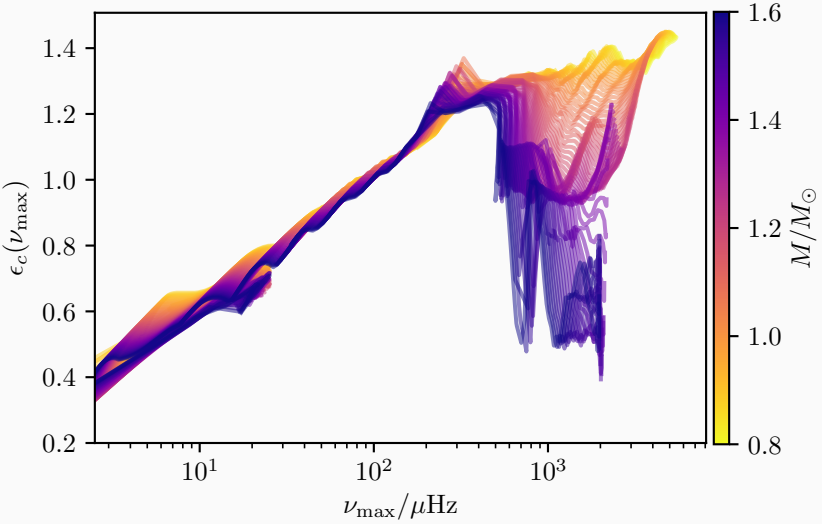

$\sqrt{V(t)/\omega^2}$
$\propto \sin \omega t$
$\psi \sim A(\omega t) j_l(\omega t - \delta)$
$\frac{\mathrm{d}\delta}{\mathrm{d}(\omega t)} = \frac{V(t)}{\omega^2} \sin^2(\omega t - \delta)$
Values for $\nu = 2500\,\mu$Hz
t/T
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
0.0
0.2
0.4
0.6
0.8
1.0


$\sqrt{V(t)t^2}$
$\mathrm{sinc}\,(\omega t - \delta)$
$\left(1 - \frac{\delta}{\omega t}\right)$
$\frac{\mathrm{d}\delta}{\mathrm{d}(\omega t)} = V(t)t^2\,\mathrm{sinc}^2\,(\omega t - \delta)\left(1 - \frac{\delta}{\omega t}\right)^2$
Values for $\nu = 2500\,\mu$Hz
t/T
8
7
6
5
4
3
2
1
0
−1
0.0
0.1
0.2
0.3
0.4
0.5

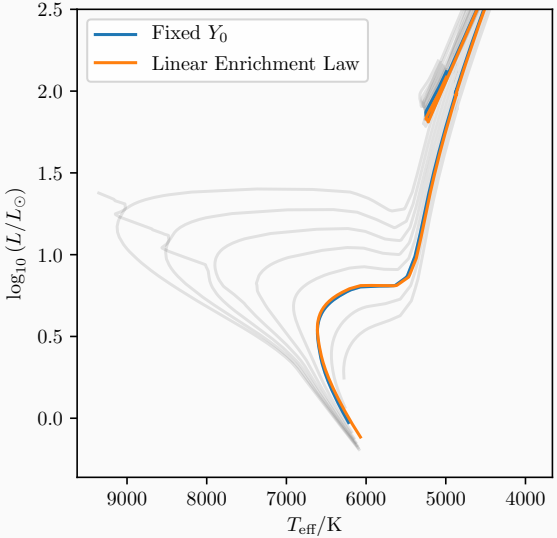
Structural phase offsets

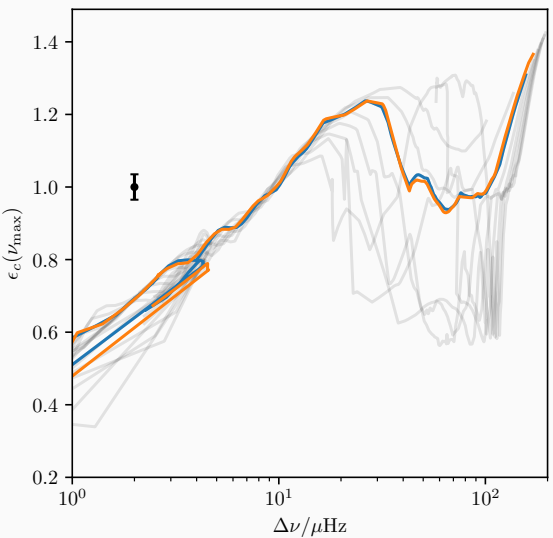


$\epsilon - \nu_{\max}$ Diagram

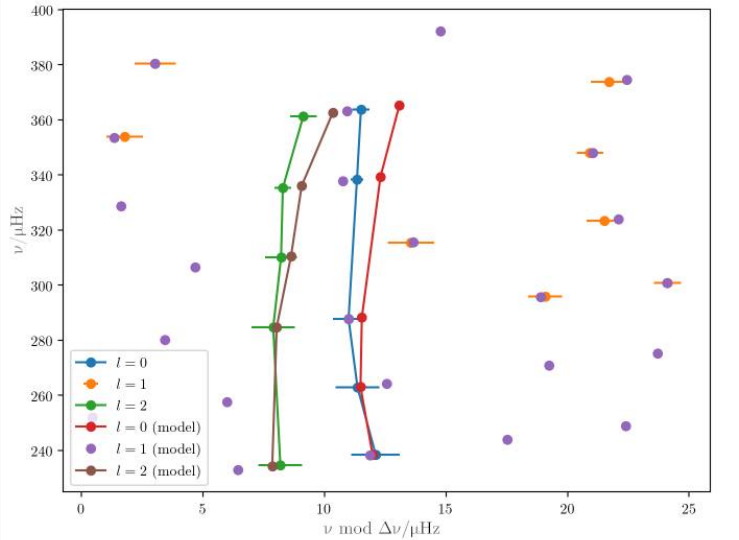


Enrichment Law ($\frac{\mathrm{d}Y_0}{\mathrm{d}Z_0}$)





Surface effects



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