Yale

Deviations from the Scaling Relationship of the Large Frequency Separation

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Joel Ong

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Department of Astronomy, Yale University

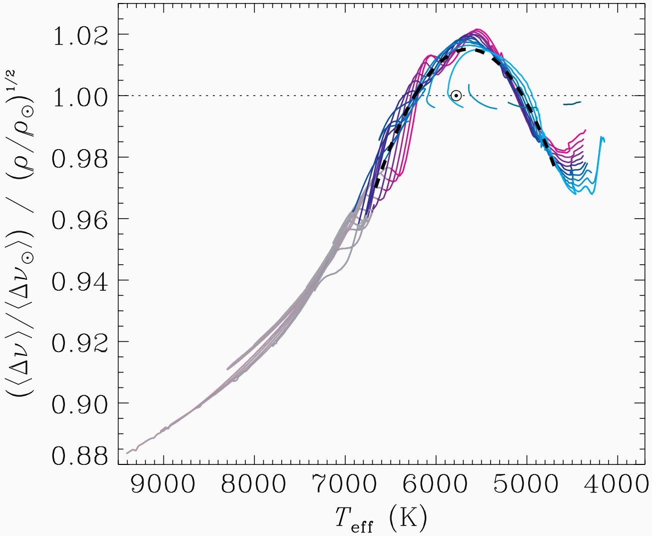


Figure 1: from White et al. (2011)

Describing scaling deviations

Extant parameterisations only fit main body of curve:

- Parabola (White et al., 2011)
- Damped sinusoid (Guggenberger et al., 2016)
- Empirical fit from symbolic regression (Guggenberger et al., 2017)

Alternative approaches are purely empirical:

- Recourse to grid, $f_{\Delta\nu}$ (Sharma et al., 2016)
- Extra terms for glitches (Kallinger et al., 2018)

Scaling vs asymptotics

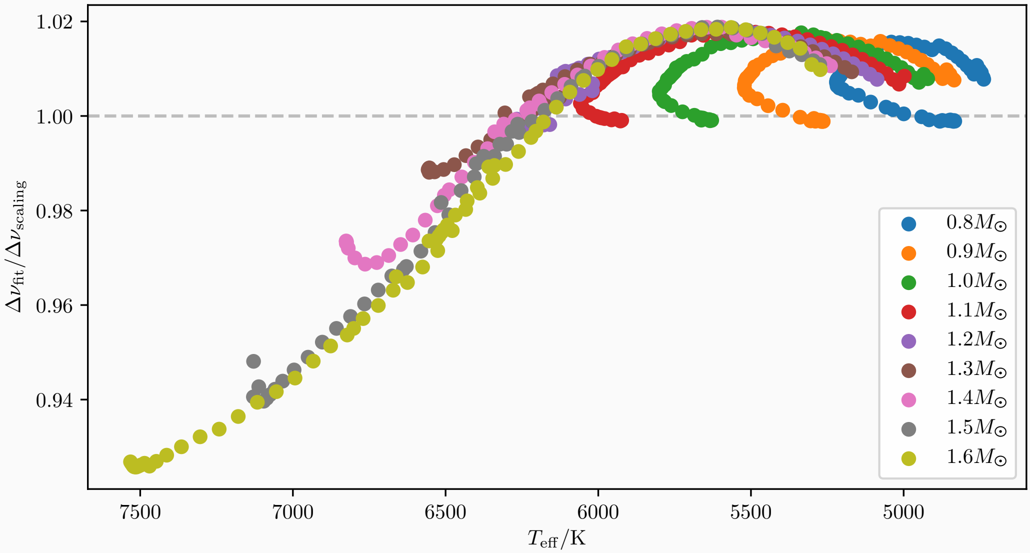
$\Delta\nu$ emerges from asymptotic expression for frequencies (Tassoul, 1990)

$$\nu_{nl} \sim \frac{1}{2T_0}\left(n + \frac{l}{2} + \alpha\right) \implies \Delta\nu \sim \frac{1}{2T_0}.$$

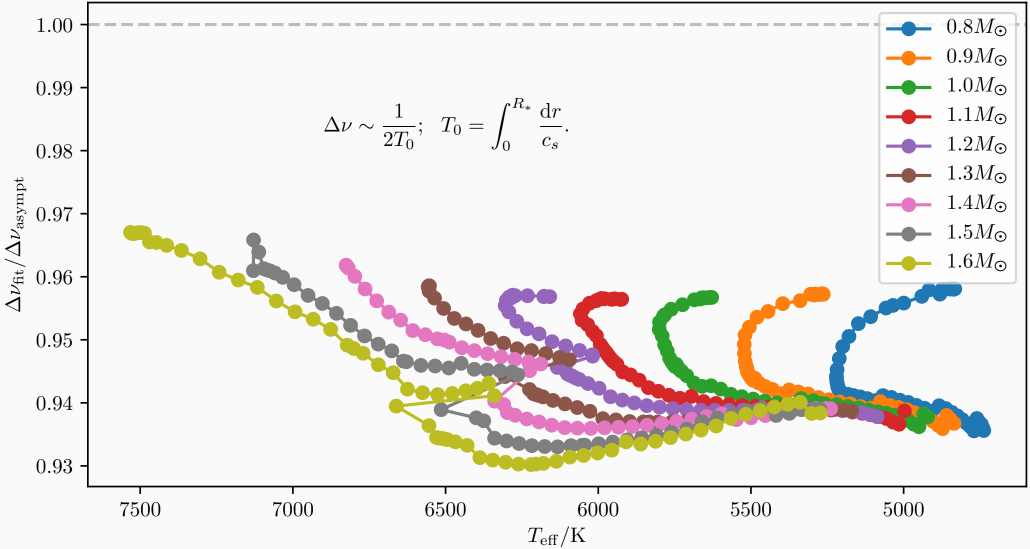
Assuming hydrostatic equilibrium, the sound-crossing time scales with the dynamical time, so

$$\boxed{\Delta\nu \sim \sqrt{G\rho}}.$$

Scaling deviations



Asymptotic deviations



Differentiating the JWKB expression for radial modes (Deubner and Gough, 1984)

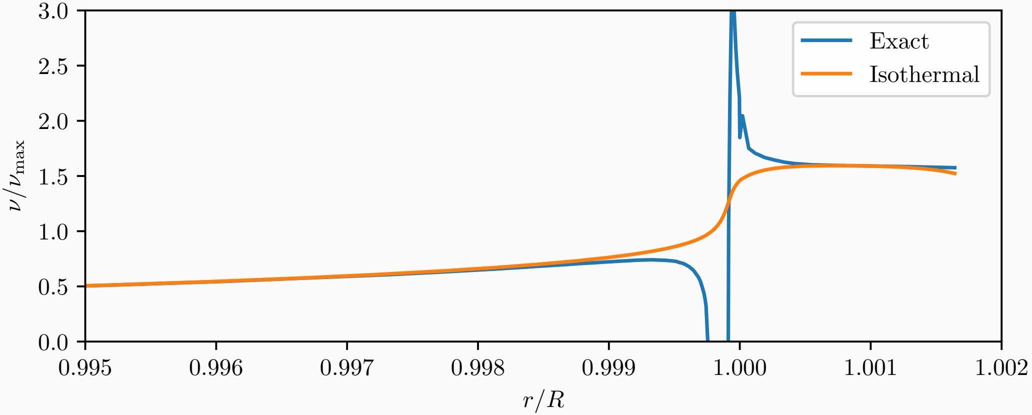
$$\omega \int_{r_1}^{r_2} \frac{\mathrm{d}r}{c_s} \sqrt{1 - \frac{\omega_{\mathrm{ac}}^2}{\omega^2} - \frac{S_l^2}{\omega^2}\left(1 - \frac{N^2}{\omega^2}\right)} \sim \omega \int_{r_1}^{r_2} \sqrt{1 - \frac{\omega_{\mathrm{ac}}^2}{\omega^2}} \frac{\mathrm{d}r}{c_s} \sim \pi (n - \alpha)$$

yields expression for first differences (neglecting $\frac{\partial \alpha}{\partial \nu}$) as

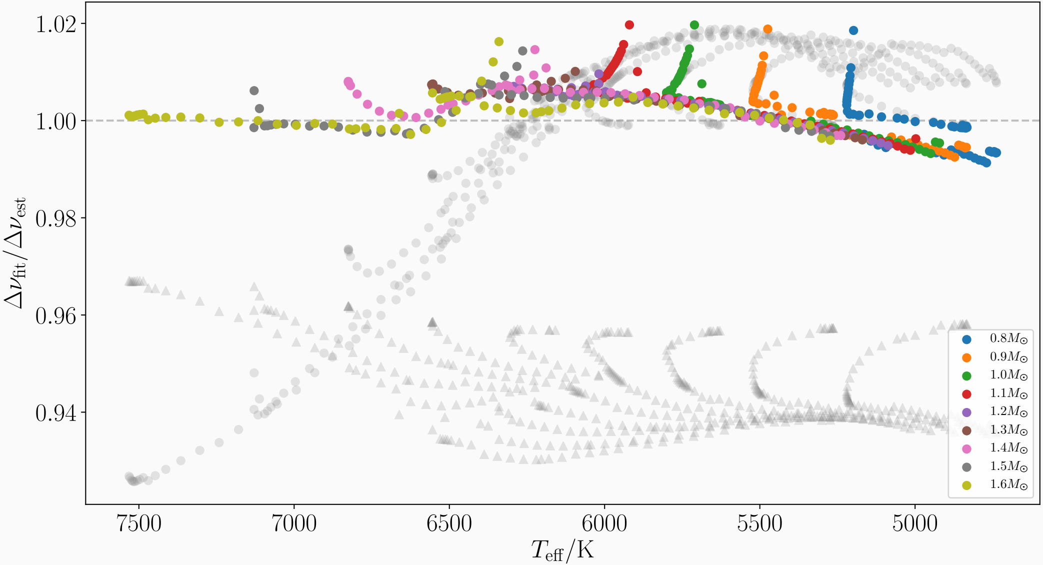
$$\boxed{\Delta\nu \sim \left(2 \int_{r_1}^{r_2} \frac{\mathrm{d}r}{c_s} \frac{1}{\sqrt{1 - \frac{\omega_{\mathrm{ac}}^2}{\omega^2}}}\right)^{-1}}$$

Numerical Experiment

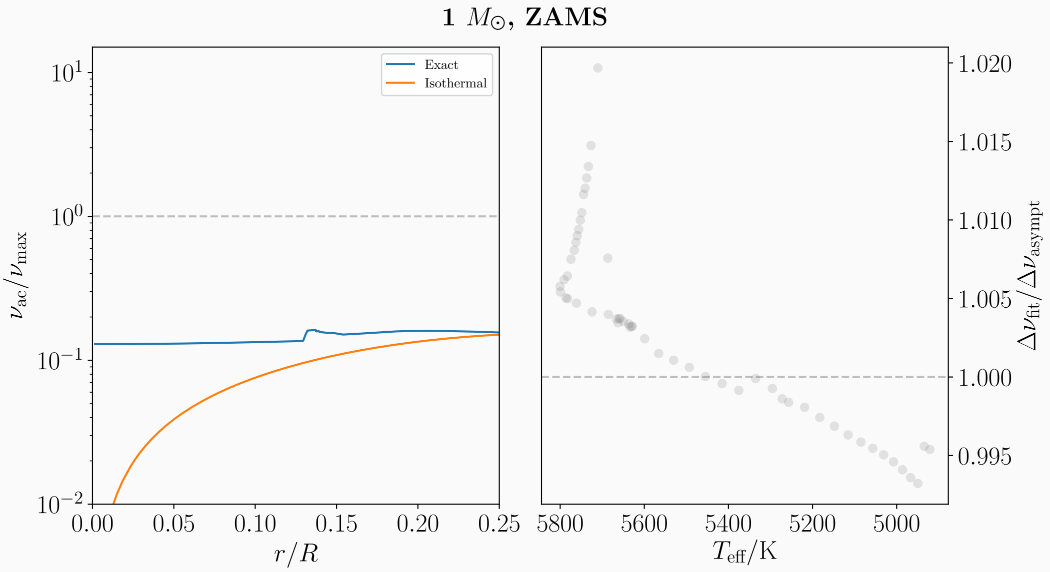
- MESA models with solar-calibrated α_{MLT} and Y_0 at solar metallicity
- All expressions evaluated at ν_{max} for radial modes (frequencies for fit from GYRE)
- Isothermal approximation for the acoustic cutoff frequency ω_{ac}



Modified asymptotic deviations: Main Sequence

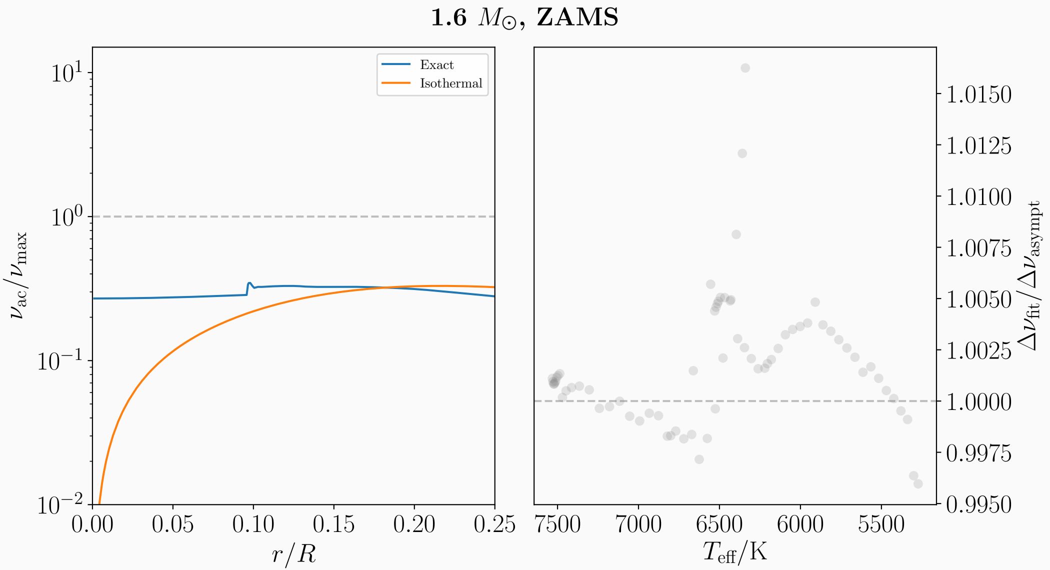


Why the bump?



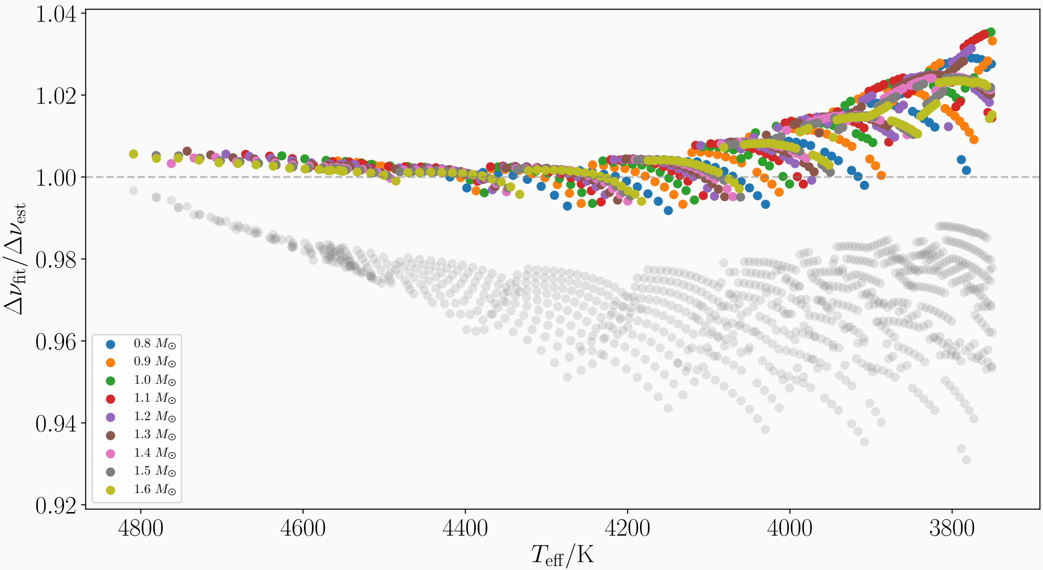
1 $M_\odot$, ZAMS
Exact
Isothermal
$\nu_{\rm ac}/\nu_{\rm max}$
10^1
10^0
10^{-1}
10^{-2}
0.00
0.05
0.10
0.15
0.20
0.25
r/R
5800
5600
5400
5200
5000
$T_{\rm eff}/{\rm K}$
1.020
1.015
1.010
1.005
1.000
0.995
$\Delta\nu_{\rm fit}/\Delta\nu_{\rm asympt}$

Why the bump?

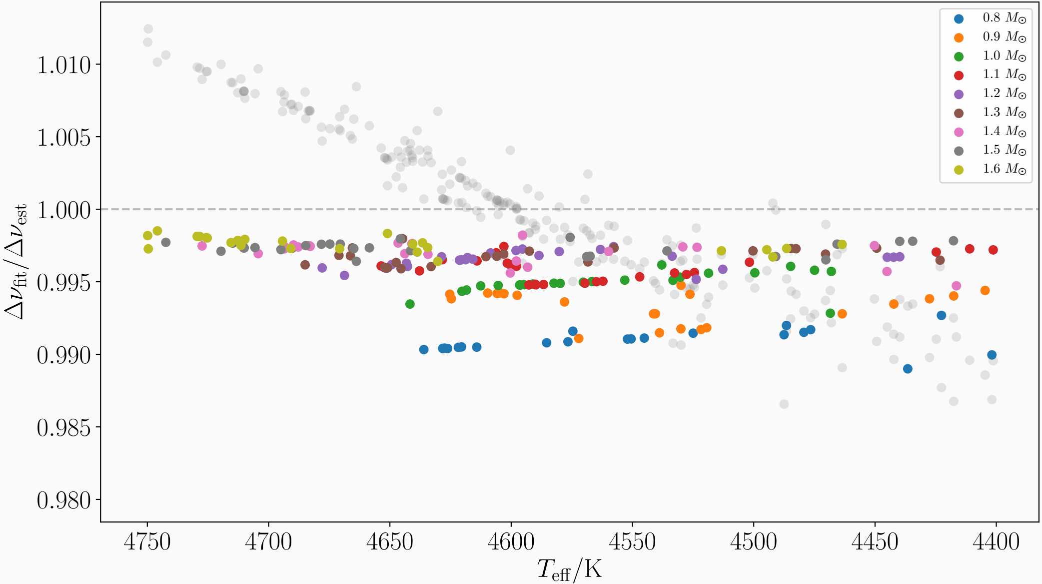


1.6 $M_\odot$, ZAMS
Exact
Isothermal
10^{1}
10^{0}
10^{-1}
10^{-2}
ν_{ac}/ν_{max}
0.00
0.05
0.10
0.15
0.20
0.25
r/R
7500
7000
6500
6000
5500
T_{eff}/K
1.0150
1.0125
1.0100
1.0075
1.0050
1.0025
1.0000
0.9975
0.9950
$\Delta\nu_{fit}/\Delta\nu_{asympt}$

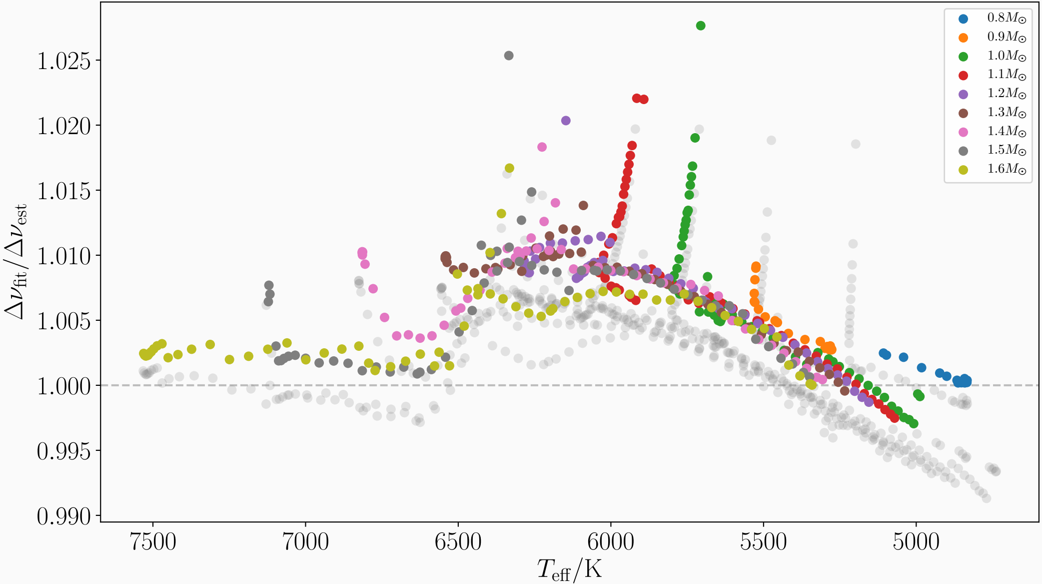
Modified asymptotic deviations: RGB



Modified asymptotic deviations: HB



Only minor dependence on model atmosphere



Conclusions

- A single asymptotic expression quite robustly describes the radial large frequency separation of stellar models of various masses and evolutionary stages to within about 1% accuracy:

$$\boxed{\Delta\nu \sim \left(2 \int_{r_1}^{r_2} \frac{\mathrm{d}r}{c_s} \frac{1}{\sqrt{1 - \frac{\omega_{\mathrm{ac}}^2}{\omega^2}}} \right)^{-1}}$$

- Remaining deviations exhibit minor dependence on model physics and stellar properties
  - Bifurcation point: qualitative change in lower turning point of mode cavity during MS turnoff

MESA models

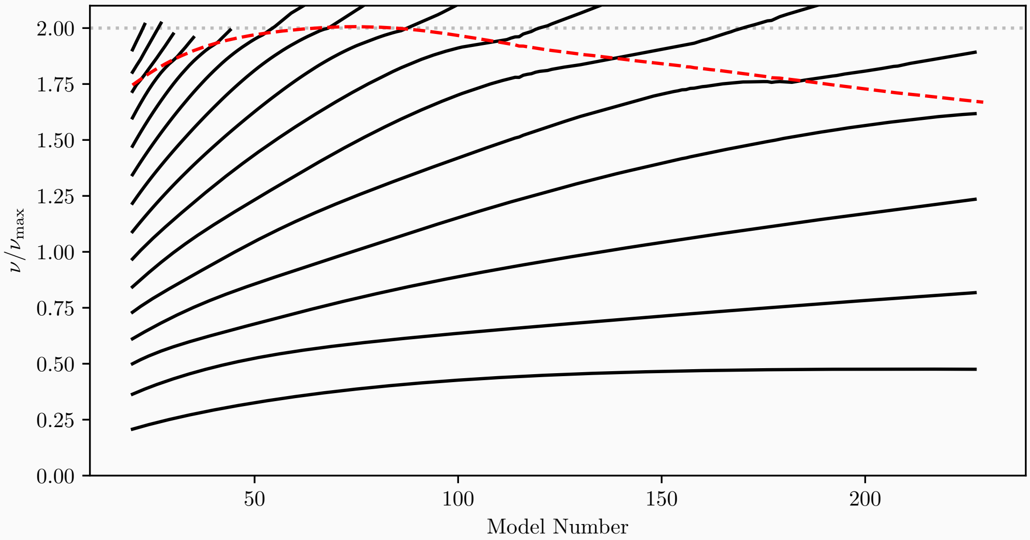
Current MESA models:

- MESA version 10108
- With overshoot and diffusion (default MESA values)
- GS98 opacities
- No mass loss for evolution to ZAHB

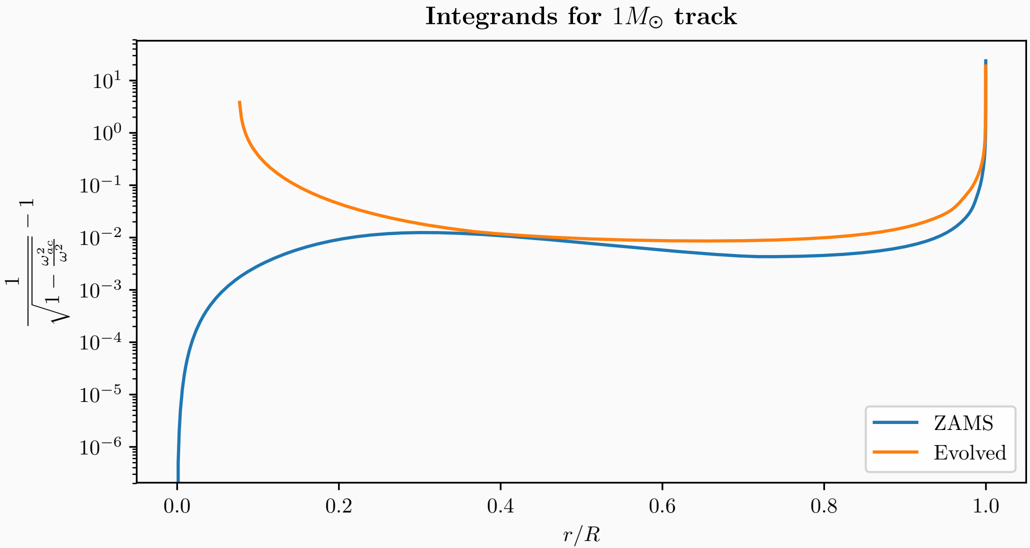
`todo.txt`

- Mass loss
- MESA v10398
- Metallicities other than solar
- Parabolic cylinder function
- Vary overshoot and/or diffusion
- Write paper
- Procrastinate

Why the terraces?



Integrable singularities



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